RANKING PERFORMANCE MEASURES WHEN CONTRACTS ARE RENEGOTIATED[*]

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Abstract

This paper employs a dynamic LEN agency model with renegotiation. First, I provide sufficient conditions in terms of the likelihood ratios of the performance measures for ranking information systems, both for implementing exogenously given actions and on the equilibrium path. Second, I characterize sufficient conditions so there are no losses from contract renegotiation; in particular, I show that conditional controllability of second-period performance with respect to first period effort is necessary for losses from renegotiation to occur. Third, I characterize a renegotiation sufficient statistic condition that is necessary and sufficient for additional information to have no value.

KEYWORDS: likelihood functions, multiple tasks, renegotiation, LEN.

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1 Introduction

Following the seminal work of Blackwell (1951), see also Marschak and Myiasawa (1968), a fruitful avenue of research in accounting has been the effort to understand the value of particular features of the performance measurement provided by accounting in different decision settings of interest. One of the frameworks usually employed consists of three ingredients: an accounting information system that provides performance reports (earnings, managerial performance evaluations), a decision problem (investment, operating decisions), and an institutional setting that links managerial compensation and the performance reports (incentive contracts, career concerns). The cornerstone of the analysis is that the information is used optimally: that means both the decision problem and the use of information to provide incentives are simultaneously endogenously determined. Thus, the insights obtained on the value of information stand on two legs: determining the optimal decision and efficiently using the information to induce the optimal decision.

As the settings of interest become increasingly complex (multiple periods, multiple tasks, contract renegotiation, earnings management to name a few ingredients), both the optimal decision and the efficient use of information increase in complexity making it harder to understand all the interactions that determine the value of accounting information. Separating the determination of the optimal decision from the efficient use of information, to the extent possible, simplifies the analysis by separating the two roles played by performance information. Moreover, the distribution of information and decision rights in large decentralized organizations makes a partial equilibrium analysis (where decisions are exogenously specified) more relevant.

In this paper, I examine the ranking of performance measures in a two-period agency with contract renegotiation. For tractability I employ a LEN model. First, I determine sufficient conditions for ranking performance measures both when the agent's actions are exogenously determined and when they are endogenously determined. Second, I use the ranking conditions to examine under what circumstances there is a loss to the principal due to contract renegotiation. In particular, the controllability of second-period performance information with respect to the first-period action, conditional on first-period performance is necessary for losses from renegotiation. An alternative interpretation is that losses from renegotiation occur only when not all the performance information pertaining to the first-period action is timely (are reported prior to renegotiation). Third, I characterize a renegotiation sufficient statistic condition that is necessary and sufficient for additional performance information to have no value.

The paper contributes to our understanding of how contract renegotiation, a feature of the institutional setting, affects the ranking of performance measures, that is the value of information. Renegotiation changes both how information is used efficiently ex-post and the optimal actions to be induced. By separating the efficient contracts from the optimal actions, I highlight the different effects of renegotiation on the two. Sometimes, renegotiation can reduce (to zero) the value of a performance measure, other times it does not change the value of a performance measure. In the latter case, renegotiation only distorts ex-post the choice for the optimal agent action.

The ranking conditions for performance measures under contract renegotiation are, to the best of my knowledge, the first in the literature. For single-period, single-task pure moral hazard models, a ranking of performance measures is provided by Kim (1995) and Kim and Suh (1991). For single-period, multi-task pure moral hazard models, a ranking of performance measures is provided by Christensen, Şabac, and Tian (2010). The paper extends the analysis in Christensen et al. (2010) to a dynamic agency with renegotiation (within a LEN multi-task model). I develop a VCM ranking condition similar to that in Christensen et al. (2010) both for implementing exogenously chosen actions and for implementing the optimal actions.

The characterization of conditions under which there is no loss from renegotiation is a step towards better understanding what drives the losses from renegotiation. Here, a distinction is made between cases when renegotiation effectively prevents the principal from using nontimely performance information and cases when renegotiation of contracts alone has no effect on the performance information the principal can use, but distorts the choice of optimal actions to be induced in equilibrium. The former case is illustrated by renegotiation in the model of Fudenberg and Tirole (1990); the latter case is illustrated by single-task, single-performance measure LEN models with renegotiation such as Gibbons and Murphy (1992), Indjejikian and Nanda (1999), Christensen, Feltham, and Şabac (2003, 2005), Şabac (2007, 2008).

The timeliness of performance relative renegotiation is expressed in terms of conditional controllability; the necessity of conditional controllability of second-period performance with respect to first-period effort for losses from renegotiation extends the conditional controllability principle of Antle and Demski (1988). If the second-period performance information is not conditionally controllable with respect to first-period effort, than it has no value in providing incentives for first-period effort and, thus, there is no loss from renegotiation because all the performance information pertaining to first-period effort is timely.

The renegotiation sufficient statistic condition that is necessary and sufficient for additional information to have no value under renegotiation is also, to the best of my knowledge, the first in the literature. The renegotiation sufficient statistic condition is neither implied, nor it implies the usual sufficient statistic condition that is necessary and sufficient for additional information to have no value in a single-period agency, see Holmström (1979).

2 The dynamic agency model

The model in this section is a two-period version of the single-period multi-task LEN model in Holmström and Milgrom (1991) and Feltham and Xie (1994). A risk neutral principal owns a production technology that requires productive effort a_t from a risk and effort averse agent in two successive periods, $t = 1, 2$; there are m_t tasks $a_t = (a_{t1}, a_{t2}, \ldots, a_{tm_t}) \in \mathbb{R}^{m_t}$ in each period.[1] The principal's expected benefit of agent effort is a weakly concave function $b(a)$ of the agent's action. The agent has exponential utility of consumption c with multiplicatively separable effort cost $u(c) = -\exp(-\,r[c - \kappa(a_1) - \kappa(a_2)])$, where $\kappa(a_t)$ is a strictly convex function of the agent's action.[2]

After the agent chooses action a_t, a contractible set of performance measures y_t is reported; there are n_t performance measures $y_t = (y_{t1}, \ldots, y_{tn_t})$ in each period, $t = 1, 2$. I assume a general structure in which the first-period action may impact second-period performance, and the noise terms are correlated across performance measures both within periods and across periods,

$$\begin{aligned} y_1 &= M_{11} a_1 + \varepsilon_1 \\ y_2 &= M_{21} a_1 + M_{22} a_2 + \varepsilon_2\ . \end{aligned} \tag{1}$$

The performance measures are joint normally distributed and the noise terms are independent of the agent's actions. Using matrix notation, $y = Ma + \varepsilon$, where $y = (y_1, y_2)$, $a = (a_1, a_2)$, $\varepsilon = (\varepsilon_1, \varepsilon_2)$. The sensitivity matrix M and the variance-covariance matrix of the noise vector ε are partitioned as

$$M = \begin{bmatrix} M_{11} & 0 \\ M_{21} & M_{22} \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}. \tag{2}$$

There are m_t tasks and n_t performance measures in each period. Thus, $M_{tt'}$ is an $n_t \times m_{t'}$

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[1] In what follows, I use the following vector and matrix algebra notation. Vectors are thought of as column vectors in all cases, and the scalar product of two vectors a and b in $\mathbb{R}^m$ is denoted by $a \cdot b$. For a matrix $M = [m_{ij}]_{1\le i\le n, 1\le j\le m}$ with n rows and m columns, I do not distinguish between the matrix and the associated linear operator $M : \mathbb{R}^m \longrightarrow \mathbb{R}^n$ defined by $Mb = (\sum_{1\le j\le m} m_{ij} b_j)_{1\le i\le n}$. As a consequence, I denote the transpose matrix as M^*, the same as the adjoint operator. Throughout the paper I use $A^*a \cdot b = a \cdot Ab$ and $(AB)^* = B^*A^*$.

[2] For simplicity, I assume a single consumption date and no discounting; this is without loss of generality in LEN models in that it does not affect the qualitative nature of the results, see Dutta and Reichelstein (1999) and Şabac (2007, 2008). I also assume that the agent's effort cost is separable across the two periods. This representation has the further advantage of allowing an easy comparison between the renegotiation problem and the dynamic full-commitment multi-task problem because the latter is equivalent to a single-period multi-task problem.

matrix of sensitivities of the n_t performance measures y_t to managerial effort $a_{t'}$ on the $m_{t'}$ tasks in period t'. The compensation contracts are restricted to be linear, $c(y) = f + v_1 \cdot y_1 + v_2 \cdot y_2$.

The above set of assumptions has three key consequences. First, the agent's compensation is normally distributed. Second, the agent's actions do not impact the variance of compensation. Third, the agent's preferences have a mean-variance representation given by the agent's certainty equivalent

$$\mathrm{CE}_{t-1}(c(y)|a_1, a_2) = \mathrm{E}_{t-1}[c(y)|a_1, a_2] - \frac{1}{2} r \mathrm{var}_{t-1}(c(y)) - \kappa(a_1) - \kappa(a_2) \ , \tag{3}$$

where in each case the subscript $t-1$ denotes "conditional on information available at the start of period t."

Full commitment benchmark

The time line of events under full commitment is as follows. The principal makes a take-it-or-leave-it contract offer $c^I(y)$ to the agent at the start of the first period. The agent accepts the contract if its certainty equivalent is greater than or equal to the agent's reservation certainty equivalent at contracting time,

$$\mathrm{CE}_0(c^I(y)|a_1, a_2) \geq \mathrm{CE}_0 \ . \tag{4}$$

If the agent rejects the principal's offer, the principal's payoff is zero, and the agent gets his reservation certainty equivalent CE_0. If the agent accepts the contract offer, he next chooses action a_1. After the agent has chosen a_1, the first-period performance y_1 is publicly reported. The agent then chooses action a_2, the second-period performance y_2 is publicly reported, and the agent's compensation is determined by $c^I(y)$.

Because productive effort is not observable and not directly contractible, the agent's action choice maximizes his certainty equivalent of compensation, which in this case is equivalent to a_t satisfying the first-order condition (the agent's incentive compatibility constraint)

$$\nabla_{a_t} \mathrm{CE_t}(c(y)|a_1, a_2) = \nabla_{a_t} \mathrm{E_t}[c(y)|a_1, a_2] - \nabla_{a_t} \kappa(a_t) = 0 \ , \tag{5}$$

where ∇_{a_t} denotes the gradient vector of partial derivatives $(\partial/\partial a_{t1}, \ldots, \partial/\partial a_{tm})$ and $c = c^I$ is the contract that determines the agent's compensation. In writing the agent's incentive compatibility constraint, I have used the fact that the action does not affect the variance of compensation.

Renegotiation

The time-line of events under renegotiation is as follows. The principal makes a take-it-or-leave-it contract offer $c^I(y)$ to the agent at the start of the first period. The agent accepts the contract if its certainty equivalent is greater than or equal to the agent's reservation certainty equivalent at contracting time, see (4). If the agent rejects the principal's offer, the principal's payoff is zero, and the agent gets his reservation certainty equivalent CE_0. If the agent accepts the contract offer, he next chooses action a_1. After the agent has chosen a_1, the first-period performance y_1 is publicly reported. Following the report y_1, the principal makes a take-it-or-leave-it renegotiation offer $c^R(y)$ to the agent. If the renegotiation offer is accepted, it replaces the initial contract, the agent chooses action a_2, the second-period performance y_2 is publicly reported, and the agent's compensation is determined by $c^R(y)$. If the agent rejects the renegotiation offer, the initial contract remains in effect, and the agent's compensation is determined by $c^I(y)$. The agent accepts the renegotiation offer if, and only if, the continuation certainty equivalent under the renegotiation offer is at least as much as that under the initial contract,

$$\mathrm{CE}_1(c^R(y)|a_1, a_2) \geq \mathrm{CE}_1(c^I(y)|a_1, a_2) \ , \tag{6}$$

where the continuation certainty equivalent at renegotiation time for a contract $c(y)$ is

$$\mathrm{CE}_1(c(y)|a_1, a_2) = \mathrm{E}_1[c(y)|a_1, a_2] - \frac{1}{2} r \mathrm{var}_1(c(y)) - \kappa(a_1) - \kappa(a_2) \ . \tag{7}$$

The subscript 1 on the expectation and variance denotes expectation and variance conditional on information available at renegotiation time. In the agent's case, this is reported performance y_1 and past effort a_1; in the principal's case, this is reported performance y_1 and conjectured agent effort $\hat{a}_1$.

The agent's incentive compatibility constraint has the same form as above, see (5), except that $c(y)$ denotes the contract expected to determine the agent's compensation. That is $c(y) = c^I(y)$ when the renegotiation offer is rejected and $c(y) = c^R(y)$ when the renegotiation offer is accepted.

Following the same approach as Grossman and Hart (1983), I separate the principal's problem in inducing given actions a_1, a_2 with an efficient contract and determining the optimal actions to be induced in equilibrium.

In order to write the efficient contracts in terms of likelihood ratios as in Christensen et al. (2010), I need to first set some important notation. Let η denote the information system that reports performance measures y_1, y_2 and let $\phi(y_1, y_2|a_1, a_2, \eta)$ denote the joint density function of the distribution of y_1, y_2. In addition to the joint distribution of the performance measures, which represents beliefs before the reporting of y_1, I am interested in posterior beliefs following the reporting of y_1. These are characterized by the conditional density function of y_2 given y_1 and $\hat{a}_1$,

$$\phi_2(y_1, y_2|\hat{a}_1, a_2, \eta) = \phi(y_2|a_2, y_1, \hat{a}_1, \eta) = \frac{\phi(y_1, y_2|\hat{a}_1, a_2, \eta)}{\bar{\phi}_1(y_1|\hat{a}_1, \eta)} \ , \tag{8}$$

where $\hat{a}_1$ represents the conjectured first-period action chosen by the manager and $\bar{\phi}_1(y_1|\hat{a}_1, \eta)$

represents the density of the marginal distribution of y_1,[3]

$$\bar{\phi}_1(y_1|\hat{a}_1,\eta) = \int \phi(y_1,y_2|\hat{a}_1,a_2,\eta)\ dy_2\ . \tag{9}$$

The likelihood ratios of the distributions above are as follows. For the joint and for the conditional distributions,

$$\begin{aligned} L(\phi,\eta|a_1,a_2) &= (L_1(\phi,\eta|a_1,a_2), L_2(\phi,\eta|a_1,a_2)) \\ &= \left(\frac{\nabla_{a_1}\phi(y_1,y_2|a_1,a_2,\eta)}{\phi(y_1,y_2|a_1,a_2,\eta)}, \frac{\nabla_{a_2}\phi(y_1,y_2|a_1,a_2,\eta)}{\phi(y_1,y_2|a_1,a_2,\eta)}\right)\ , \end{aligned} \tag{10}$$

$$L_2(\phi,\eta|a_1,a_2) = L(\phi_2,\eta|a_1,a_2) = \frac{\nabla_{a_2}\phi_2(y_1,y_2|\hat{a}_1,a_2,\eta)}{\phi_2(y_1,y_2|\hat{a}_1,a_2,\eta)}\ . \tag{11}$$

To see that $L_2(\phi,\eta|a_1,a_2) = L(\phi_2,\eta|a_1,a_2)$ in the two definitions above, note that $L(\phi_2) = \nabla_{a_2}\phi_2/\phi_2 = \nabla_{a_2}\phi/(\bar{\phi}_1\phi/\bar{\phi}_1) = \nabla_{a_2}\phi/\phi = L_2(\phi)$ because $\phi_2(y_1,y_2) = \phi(y_1,y_2)/\bar{\phi}_1(y_1)$, and $\nabla_{a_2}\bar{\phi}_1(y_1) = 0$. For the marginal distribution,

$$\bar{L}_1(\phi,\eta|a_1) = L(\bar{\phi}_1,\eta|a_1) = \frac{\nabla_{a_1}\bar{\phi}_1(y_1|\hat{a}_1,\eta)}{\bar{\phi}_1(y_1|\hat{a}_1,\eta)}\ . \tag{12}$$

Note that the likelihood ratios in all cases are linear in the performance measures and thus are normally distributed as well.

The variance-covariance matrices of the likelihood ratios L, L_2, and $\bar{L}_1$ are denoted $\Sigma L(\phi,\eta)$, $\Sigma L_2(\phi,\eta) = \Sigma L(\phi_2,\eta)$, and $\Sigma\bar{L}_1(\phi,\eta) = \Sigma L(\bar{\phi}_1,\eta)$, respectively; the covariance of the likelihood ratios L_1, L_2 is denoted $\Sigma L_{12}(\phi,\eta) = \text{cov}(L_1(\phi|\eta), L_2(\phi,\eta))$. The notation reflects the fact that, while the likelihood ratios depend on the agent's actions, their variance-covariance matrices do not. This is because the likelihood ratios are linear in the performance measures (with coefficients independent of agent effort) and the latter have variance-covariance matrices that are also independent of agent effort.

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[3] When considering the agent's beliefs, $\hat{a}_1 = a_1$ because the agent remembers his action choice; the distinction is important for the principal's beliefs.

3 Ranking performance measures for given actions

In this section, I assume that the principal can commit to the actions he wants the agent to implement; alternatively, the agent's actions are not a choice variable for the principal but exogenous to the agency problem. In all cases, the principal seeks to induce actions (a_1, a_2) with a linear contract $c(y)$ at minimum cost.

Full commitment benchmark

In the full commitment case, at the start of the first period, the principal commits both to the actions to be induced, (a_1, a_2), and to the contract $c(y)$. Because the second-period action choice does not depend on the first-period action choice and performance report, this case is fully equivalent to the agent simultaneously choosing both actions at the same time, that is a single-period multi-task model. Thus, the efficient contract is characterized by Lemma 1 from Christensen et al. (2010) and the ranking of information systems is given by the VCM condition, Proposition 4 in Christensen et al. (2010). For completeness, both results are stated below.

Lemma 1 *Under the assumptions of the LEN model with full commitment:*

(i) For an implementable action $a = (a_1, a_2)$, an optimal multiplier on the incentive compatibility constraints (5) is denoted by $\mu = (\mu_1, \mu_2) = \Sigma L(\phi, \eta)^{-1} \nabla_a \kappa(a)$ and is given by

$$\Sigma L(\phi, \eta) \frac{\mu}{r} = \nabla_a \kappa(a) \ , \tag{13}$$

where $\nabla_a \kappa(a) = (\nabla_{a_1} \kappa(a_1), \nabla_{a_2} \kappa(a_2))$.

(ii) The set of implementable actions $A_t(\eta)$, is characterized by the row space of M and the marginal cost of effort $\nabla_a \kappa(a)$, $A(\eta) = \{a = (a_1, a_2) \in \mathbb{R}^{m_1} \times \mathbb{R}^{m_2} | \nabla_a \kappa(a) \in \mathrm{Im}(M^*)\}$.

(iii) For any $(a_1, a_2) \in A(\eta)$, the efficient linear contract that implements a_1, a_2 is charac-

terized by

$$
\begin{aligned}
c^R(y) = \mathrm{CE}_0 + \kappa(a_1) + \kappa(a_2) + \frac{1}{2} r \Sigma L(\phi,\eta)^{-1} \nabla_a \kappa(a) \cdot \nabla_a \kappa(a) \\
+ \Sigma L(\phi,\eta)^{-1} \nabla_a \kappa(a) \cdot L(\phi,\eta | a_1, a_2)\ .
\end{aligned} \tag{14}
$$

The contract is uniquely determined, even when the multiplier μ that satisfies (13) *is not.*

Proposition 1 *Under the LEN assumptions with full commitment for two information systems η^1 and η^2, if the row spaces of M_1 and M_2 are the same, the strict VCM condition $\Sigma L(\phi,\eta^2) > \Sigma L(\phi,\eta^1)$ is sufficient for the principal to strictly prefer η^2 to η^1 for implementing any action $a \in A(\eta^1) = A(\eta^2)$. The weak VCM condition is sufficient for a weak preference of η^2 over η^1.*

Renegotiation

Next, I characterize the efficient contract that induces action a_t in period $t = 1, 2$, subject to renegotiation at the start of the second period. At the start of the first period, the principal commits to the actions to be induced (a_1, a_2) but cannot commit to the initial contract c^I; the principal and the agent can renegotiate the contract following the reporting of the first-period performance information. Proceeding by backwards induction, I start with the second period. Renegotiation means that the contract is efficient conditional on information available at the start of the second period.

Applying Lemma 1 from Christensen et al. (2010) at renegotiation time gives the principal's efficient renegotiation offer that implements a given action and is accepted by the agent.

Lemma 2 *Under the assumptions of the LEN model with renegotiation:*

(i) For an implementable action a_2, an optimal multiplier on the incentive compatibility constraint (5) in period $t = 2$ is denoted by $\mu_2 = \Sigma L_2(\phi,\eta)^{-1} \nabla_{a_2} \kappa(a_2)$ and is characterized

by

$$\Sigma L_2(\phi,\eta)\frac{\mu_2}{r} = \nabla_{a_2}\kappa(a_2)\ . \tag{15}$$

(ii) The set of second-period implementable actions $A_2(\eta)$ *is characterized by the row space of* M_{22} *and the cost of effort* $\kappa(a_2)$, $A_2(\eta) = \{a_2 \in \mathbb{R}^m | \nabla_{a_2}\kappa(a_2) \in \mathrm{Im}(M_{22}^*)\}$.

(iii) For any $a_2 \in A_2(\eta)$, *the efficient linear contract that implements* a_2 *is characterized by*

$$\begin{aligned} c^R(y) = \mathrm{CE}_1(c^I(y_1,y_2)|\hat{a}_1,a_2) + \kappa(\hat{a}_1) + \kappa(a_2) + \frac{1}{2}r\Sigma L_2(\phi,\eta)^{-1}\nabla_{a_2}\kappa(a_2)\cdot\nabla_{a_2}\kappa(a_2) \\ +\Sigma L_2(\phi,\eta)^{-1}\nabla_{a_2}\kappa(a_2)\cdot L_2(\phi,\eta|a_1,a_2)\ . \end{aligned} \tag{16}$$

The contract is uniquely determined, even when the multiplier μ_2 *that satisfies* (15) *is not.*

Note that (16) implies that any efficient renegotiation offer that implements a_2 and is accepted by the agent is of the form

$$c(y_1,y_2) = c_1(y_1) + \Sigma L_2(\phi,\eta)^{-1}\nabla_{a_2}\kappa(a_2)\cdot L_2(\phi,\eta|a_1,a_2)\ , \tag{17}$$

where $c_1(y_1)$ is a linear function in y_1 that depends on the initial contract offer and the agent's conjectured first-period action. Moreover, any contract of the form (17) is renegotiation-proof if offered as an initial contract at the start of the first period. Indeed, the certainty equivalent at renegotiation time of a contract of the form (17) is

$$\begin{aligned} \mathrm{CE}_1(c(y_1,y_2)|\hat{a}_1,a_2) &= c_1(y_1) - \kappa(\hat{a}_1) - \kappa(a_2) \\ &\quad - \frac{1}{2}\Sigma L_2(\phi,\eta)\Sigma L_2^{-1}(\phi,\eta)\nabla_{a_2}\kappa(a_2)\cdot\Sigma L_2^{-1}(\phi,\eta)\nabla_{a_2}\kappa(a_2) \\ &= c_1(y_1) - \kappa(\hat{a}_1) - \kappa(a_2) - \frac{1}{2}r\Sigma L_2(\phi,\eta)^{-1}\nabla_{a_2}\kappa(a_2)\cdot\nabla_{a_2}\kappa(a_2)\ . \end{aligned} \tag{18}$$

Substituting $c^I(y_1,y_2) = c(y_1,y_2)$ given by (17) in the efficient renegotiation offer (16) gives $c^R(y_1,y_2) = c(y_1,y_2) = c^I(y_1,y_2)$ and proves the claim.

Because the agent's incentives are determined by his terminal wealth and that is in turn determined by the renegotiation offer (16), it follows that any efficient sequence of contracts (c^I, c^R) subject to renegotiation can be substituted by a renegotiation-proof contract of the form in (17) that implements the same actions at the same expected cost to the principal. It is therefore without loss of generality to restrict the analysis to renegotiation-proof contracts and the renegotiation-proofness constraint is given by (17).

The additive separability obtained in (17) for the renegotiation-proof contract simplifies the analysis in two significant ways. First, to determine an efficient renegotiation-proof contract, it suffices to determine $c_1(y_1)$. This is a linear contract in y_1, and I can follow a similar procedure to that used by Christensen et al. (2010) in the single-period case. Second, the two parts of the contract are stochastically independent. Indeed, for any vector μ,

$$\begin{aligned}\iint c_1(y_1)\mu \cdot L_2(\phi)\phi(y_1,y_2)\ dy_1dy_2 &= \iint c_1(y_1)\mu \cdot \nabla_{a_2}\phi(y_1,y_2)\ dy_1dy_2\\ = \int c_1(y_1)\left(\int \mu \cdot \nabla_{a_2}\phi(y_1,y_2)\ dy_2\right) dy_1 &= \int c_1(y_1)\left(\mu \cdot \int \nabla_{a_2}\phi(y_1,y_2)\ dy_2\right) dy_1 \\ = \int c_1(y_1)\mu \cdot \nabla_{a_2}\bar{\phi}_1(y_1)\ dy_1 = 0\ .\end{aligned} \tag{19}$$

The renegotiation-proofness constraint (17) sets part of the contract given the chosen second-period action, so the principal's problem at the initial contract date is to optimize over $c_1(y_1)$ in order to induce the chosen first-period action a_1:

$$\min_{c_1(y_1)\in C} \mathrm{E}[c(y_1,y_2)|a_1,a_2]\ , \tag{20}$$

subject to the agent's participation constraint (4), the incentive compatibility constraint

$$\nabla_{a_1}\mathrm{CE}_0(c(y)|a_1,a_2) = \nabla_{a_1}\mathrm{E}(c(y_1,y_2)|a_1,a_2) - \nabla_{a_1}\kappa(a_1) = 0\ , \tag{21}$$

and the renegotiation-proofness constraint (17). Note that the renegotiation-proofness constraint also ensures the induced second-period action is a_2.

The agent's certainty equivalent conditional on anticipated action choice a_1 and on the principal's conjecture $\hat{a}_1$ is

$$\begin{aligned}\mathrm{CE}_0(c(y_1,y_2)|a_1,\hat{a}_1) = \int c_1(y_1)\bar{\phi}_1(y_1|a_1,\eta)\ dy_1 - \kappa(a_1) - \kappa(a_2) \\ + \Sigma L_2(\phi,\eta)^{-1}\nabla_{a_2}\kappa(a_2)\cdot \iint L_2(\phi,\eta|\hat{a}_1,a_2)\phi(y_1,y_2|a_1,a_2,\eta)\ dy_1 dy_2 \\ - \frac{1}{2}r\mathrm{var}(c_1(y_1)) - \frac{1}{2}r\mathrm{var}\left(\Sigma L_2(\phi,\eta)^{-1}\nabla_{a_2}\kappa(a_2)\cdot L_2(\phi,\eta|\hat{a}_1,a_2)\right)\ . \quad (22)\end{aligned}$$

In equilibrium, the agent's action choice a_1 coincides with the principal's conjecture, so that the double integral on the second line in (22) is equal to zero. In addition, because $\mathrm{var}(\mu\cdot L_2(\phi,\eta|\hat{a}_1,a_2)) = \mathrm{var}_1(\mu\cdot L_2(\phi,\eta|\hat{a}_1,a_2)) = \Sigma L_2(\phi,\eta)\mu\cdot\mu$, the agent's contract acceptance constraint becomes

$$\begin{aligned}\int c_1(y_1)\bar{\phi}_1(y_1|a_1,\eta)\ dy_1 - \kappa(a_1) - \kappa(a_2) - \frac{1}{2}r\mathrm{var}(c_1(y_1)) \\ - \frac{1}{2}r\Sigma L_2(\phi,\eta)^{-1}\nabla_{a_2}\kappa(a_2)\cdot\nabla_{a_2}\kappa(a_2) \geq \mathrm{CE}_0\ . \quad (23)\end{aligned}$$

The agent's incentive compatibility constraint follows from first differentiating his certainty equivalent (22) conditional on choosing action a_1 and on the principal's conjecture $\hat{a}_1$, and then setting $a_1 = \hat{a}_1$, [4]

$$\begin{aligned}\int c_1(y_1)\nabla_{a_1}\bar{\phi}_1(y_1|a_1,\eta)\ dy_1 \\ + \mathrm{cov}(L_1(\phi,\eta|a_1), L_2(\phi,\eta|a_2))\Sigma L_2(\phi,\eta)^{-1}\nabla_{a_2}\kappa(a_2) - \nabla_{a_1}\kappa(a_1) = 0\ . \quad (24)\end{aligned}$$

To see this, differentiate the double integral in the middle line of (22), which is not zero for $a_1 \neq \hat{a}_1$, and then set $a_1 = \hat{a}_1$. Denote by $l_{ti}(\phi|a_1,a_2) = \partial_{a_{ti}}\phi/\phi$ the ith component of the likelihood function in each case evaluated at a_1, a_2. The key step is as follows (suppressing

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[4] Here it important to distinguish between the agent's action choice and the principal's conjecture because only the first is a choice variable for the agent. In equilibrium the principal's conjecture is correct, and we no longer need the distinction in the incentive compatibility constraint

the η from the notation):

$$\begin{aligned}\partial_{a_{1i}} \iint l_{2j}(\phi|\hat{a}_1, a_2)\phi(y_1, y_2|a_1, a_2)\ dy_1 dy_2 &= \iint l_{2j}(\phi|\hat{a}_1, a_2)\partial_{a_{1i}}\phi(y_1, y_2|a_1, a_2)\ dy_1 dy_2 \\ &= \iint l_{2j}(\phi|\hat{a}_1, a_2) l_{1i}(\phi|a_1, a_2)\phi(y_1, y_2|a_1, a_2)\ dy_1 dy_2, \quad (25)\end{aligned}$$

and the last line is $\mathrm{cov}(l_{1i}(\phi|a_1, a_2), l_{2j}(\phi|\hat{a}_1, a_2))$.

From this point on, to simplify the notation, I denote by $A = \Sigma\bar{L}_1(\phi, \eta)^{-1}$, $C = \Sigma L_2(\phi, \eta)^{-1}$, and $B = \Sigma L_{12}(\phi, \eta) = \mathrm{cov}(L_1(\phi, \eta), L_2(\phi, \eta))$. With this notation, I can rewrite the agent's participation constraint (23) as

$$\begin{aligned}\int c_1(y_1)\bar{\phi}_1(y_1)\ dy_1 - \frac{1}{2} r \int (c_1(y_1) - \mathrm{E}[c_1(y_1)])^2 \bar{\phi}_1(y_1|a_1, \eta)\ dy_1 \\ - \frac{1}{2} r C \nabla_{a_2}\kappa(a_2) \cdot \nabla_{a_2}\kappa(a_2) - \kappa(a_1) - \kappa(a_2) \geq \mathrm{CE}_0\ , \quad (26)\end{aligned}$$

and the agent's incentive compatibility constraint (24) as

$$\int c_1(y_1) L_1(\bar{\phi}_1, \eta|a_1)\bar{\phi}_1(y_1|a_1, \eta)\ dy_1 + BC\nabla_{a_2}\kappa(a_2) - \nabla_{a_1}\kappa(a_1) = 0\ . \quad (27)$$

At this point I can apply again Lemma 1 from Christensen et al. (2010) (with an appropriately adjusted proof for the participation and incentive compatibility constraints (26) and (27)). This is possible because the renegotiation constraint (17) separates the contract in two independent parts, $c_1(y_1)$ and $C\nabla_{a_2}\kappa(a_2) \cdot L_2(\phi, \eta|a_1, a_2)$; the latter enters the participation and incentive compatibility constraints as constants independent of $c_1(y_1)$. Thus, the first part of the contract (which depends only on y_1) can be determined as in a single-period problem.

Lemma 3 *Under the assumptions of the LEN model with renegotiation:*
(i) For an implementable action a_1, an optimal multiplier on the incentive compatibility constraint (27) in period $t = 1$ is denoted by $\mu_1 = A(\nabla_{a_1}\kappa(a_1) - BC\nabla_{a_2}\kappa(a_2))$ and is

characterized by

$$\Sigma \bar{L}_1(\phi, \eta)\frac{\mu_1}{r} = \nabla_{a_1}\kappa(a_1) - BC\nabla_{a_2}\kappa(a_2)\ . \tag{28}$$

(ii) The set of first-period implementable actions $A_1(\eta, a_2)$ *is characterized by the row space of* M_{11}*, the cost of effort* $\kappa(a_1)$*, and the second-period effort* a_2*,* $A_1(\eta, a_2) = \{a_1 \in \mathbb{R}^m | \nabla_{a_1}\kappa(a_1) - BC\nabla_{a_2}\kappa(a_2) \in \mathrm{Im}(M_{11}^*)\}$*.*

(iii) For any $a_1 \in A_1(\eta, a_2)$*, the efficient linear contract* $c_1(y_1)$ *that implements* a_1*, conditional on* a_2 *being implemented in the second period is characterized by*

$$\begin{aligned} c_1(y_1) = \mathrm{CE}_0 + \kappa(a_1) + \kappa(a_2) + \frac{1}{2}rA(\nabla_{a_1}\kappa(a_1) - BC\nabla_{a_2}\kappa(a_2)) \cdot (\nabla_{a_1}\kappa(a_1) - BC\nabla_{a_2}\kappa(a_2)) \\ + \frac{1}{2}rC\nabla_{a_2}\kappa(a_2) \cdot \nabla_{a_2}\kappa(a_2) + A(\nabla_{a_1}\kappa(a_1) - BC\nabla_{a_2}\kappa(a_2)) \cdot L_1(\bar{\phi}, \eta | a_1)\ . \end{aligned} \tag{29}$$

The contract is uniquely determined, even when the multiplier μ_1 *that satisfies* (28) *is not.*

Assuming two implementable actions, $a_2 \in A_2(\eta)$ and $a_1 \in A_1(\eta, a_2)$, the next task is to determine the principal's expected cost of implementing the two actions. From Lemma 2, (16) and Lemma 3, (29) it follows that

$$\begin{aligned} \mathrm{E}[c^R(y_1, y_2) | a_1, a_2, \eta)] = \kappa(a_1) + \kappa(a_2) \\ + \frac{1}{2}rA(v_1 - BCv_2) \cdot (v_1 - BCv_2) + \frac{1}{2}rCv_2 \cdot v_2\ , \end{aligned} \tag{30}$$

where I denoted for simplicity $v_1 = \nabla_{a_1}\kappa(a_1)$ and $v_2 = \nabla_{a_2}\kappa(a_2)$. When comparing the cost of implementing the same actions with two information systems η^1 and η^2, the effort cost $\kappa(a_t)$ is the same and so is the marginal effort cost v_t; the only difference is in the risk premium. The following proposition characterizes the risk premium for implementing an arbitrary sequence of actions and allows an explicit ranking of information systems for LEN models with renegotiation.

Proposition 2 *The principal's expected cost of implementing $a = (a_1, a_2)$ is*

$$\mathrm{E}[c^R(y_1, y_2)|a_1, a_2, \eta)] = \kappa(a_1) + \kappa(a_2) + \frac{1}{2} r R(\phi, \eta)(v_1, v_2) \cdot (v_1, v_2) \ , \tag{31}$$

where the matrix $R(\phi, \eta)$ only depends on the information system and is given by

$$R(\phi, \eta) = \begin{bmatrix} A & -ABC \\ -CB^*A & C + CB^*ABC \end{bmatrix} \ , \tag{32}$$

and (v_1, v_2) is the marginal effort cost vector.

If a sequence of actions is implementable under two information systems η^1, η^2, the principal prefers the one that allows him to implement the actions with a lower risk premium. The above proposition gives a simple sufficient condition for ranking information systems.

Corollary 1 *Under the assumptions of the dynamic LEN model with renegotiation, and for any sequence of actions $a = (a_1, a_2)$ that are implementable with both η^1 and η^2, if $R(\phi, \eta^2) < R(\phi, \eta^1)$, then the principal strictly prefers η^2 to η^1 for implementing $a = (a_1, a_2)$. A weak inequality is sufficient for a weak preference.*

Note that it is not necessary to assume that $\Sigma\bar{L}_1(\phi, \eta)$ and $\Sigma L_2(\phi, \eta)$ are invertible in the above Proposition 2 and Corollary 1. For any implementable actions (a_1, a_2), there exist multipliers μ_1, μ_2 that satisfy (28) and (15). Thus, one can define a linear operator $C = \Sigma L_2(\phi, \eta)^{-1}$ for a subspace of marginal effort cost vectors v_2 and then extend it to the entire space; a similar procedure yields an operator $A = \Sigma\bar{L}_1(\phi, \eta)$. The two operators A and C are not uniquely determined in general, but by Lemma 2 and Lemma 3, the corresponding linear contract and the principal's expected compensation cost are not affected.

The ranking of partitioned matrices is very similar to the ranking of quadratic forms. The following lemma extends a standard result for quadratic forms to partitioned matrices (see Debreu 1952).

Lemma 4 *Let Q denote a symmetric matrix partitioned as*

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{*} & Q_{22} \end{bmatrix} . \tag{33}$$

Then $Q > 0$ if, and only if, $Q_{11} > 0$ and $P = Q_{22} - Q_{12}^{*}Q_{11}^{-1}Q_{12} > 0$.

Furthermore, if $Q > 0$, then $P > 0$ and the the inverse matrix Q^{-1} is

$$Q^{-1} = \begin{bmatrix} Q_{11}^{-1}(I + Q_{12}P^{-1}Q_{21}Q_{11}^{-1}) & -Q_{11}^{-1}Q_{12}P^{-1} \\ -P^{-1}Q_{21}Q_{11}^{-1} & P^{-1} \end{bmatrix} . \tag{34}$$

A stronger result that allows a direct comparison with the full commitment case can be obtained if the matrices A, C, and $R(\phi, \eta)$ are invertible. If $R(\phi, \eta)$ is invertible, denote by $\Sigma R(\phi, \eta)$ the inverse of $R(\phi, \eta)$. The following lemma from Goldberger (1964), p. 38 gives the ranking of inverse matrices.

Lemma 5 *If $Q_1 > Q_2 > 0$ are positive definite matrices of the same order, then $Q_2^{-1} > Q_1^{-1}$.*

Proposition 2 and Lemma 5 give then a ranking of information systems in terms of the matrices $\Sigma R(\phi, \eta^i)$.

Corollary 2 *Under the assumptions of the dynamic LEN model with renegotiation, and for any sequence of actions $a = (a_1, a_2)$ that are implementable with both η^1 and η^2, if $\Sigma R(\phi, \eta^1) < \Sigma R(\phi, \eta^2)$, then the principal strictly prefers η^2 to η^1 for implementing $a = (a_1, a_2)$. A weak inequality is sufficient for a weak preference.*

The matrix $\Sigma R(\phi, \eta)$ corresponds to the variance-covariance matrix of the likelihood ratio when contracts are renegotiated. Thus, Corollary 2 extends the VCM condition to a pure moral hazard case with contract renegotiation. Under the assumptions in this paper, the full commitment benchmark is equivalent to a single-period multi-task problem, so the results in Christensen et al. (2010) apply directly here. In particular, the VCM condition for ranking information systems is based on the variance-covariance of the likelihood ratio $\Sigma L(\phi, \eta)$.

In what follows, I will assume that $\text{rank}(M_{tt}) = m_t \leq n_t$; in other words, all actions are implementable and there are at least as many performance measures as there are tasks in each period. The next result provides sufficient conditions for $R(\phi, \eta)$ to be invertible, and thus, for information systems to be ranked by $\Sigma R(\phi, \eta)$ according to Corollary 2.

Lemma 6 *If M_{tt} has rank $m_t \leq n_t$, then $\Sigma \bar{L}_1(\phi, \eta)$, $\Sigma L_2(\phi, \eta)$, and $R(\phi, \eta)$ are invertible. In addition, by definition $\Sigma R(\phi, \eta) = R(\phi, \eta)^{-1}$, and*

$$\Sigma R(\phi, \eta) = \begin{bmatrix} \Sigma \bar{L}_1(\phi, \eta) + \Sigma L_{12}(\phi, \eta) \Sigma L_2(\phi, \eta)^{-1} \Sigma L_{12}(\phi, \eta)^* & \Sigma L_{12}(\phi, \eta) \\ \Sigma L_{12}(\phi, \eta)^* & \Sigma L_2(\phi, \eta) \end{bmatrix}. \tag{35}$$

4 Ranking performance measures in equilibrium

In this section, I make the additional assumptions of the "standard" LEN model as introduced by Feltham and Xie (1994) in order to explicitly determine the equilibrium actions and payoffs and to rank information systems on the equilibrium path. The principal's expected benefit given the agent's action choice $a = (a_1, a_2)$ is $b \cdot a = b_1 \cdot a_1 + b_2 \cdot a_2$, where $b = (b_1, b_2)$. The agent's effort cost is quadratic $\kappa(a_t) = \frac{1}{2} a_t \cdot a_t$. All the other assumptions of the LEN model introduced earlier are maintained in this section. The key difference is that the agent's actions are equilibrium outcomes and are no longer exogenously specified.

Full commitment benchmark

In the full commitment case, the principal chooses at the start of the first period the optimal actions to be induced and commits to the contract for the duration of the two periods. The full commitment equilibrium is the same as that characterized by the "standard" single-period multi-task model in Feltham and Xie (1994) and Christensen et al. (2010). The optimal action, optimal linear contract, the principal's surplus and the ranking of information systems are given by Lemma 3 and Proposition 5 in Christensen et al. (2010). For completeness, the two results are stated below.

Lemma 7 *In equilibrium, for the standard LEN model with full commitment, the optimal actions induced are* $a = (a_1, a_2) = (\Sigma L(\phi, \eta) + rI)^{-1}\Sigma L(\phi, \eta)b$*, the optimal linear contract is*

$$c(y) = \mathrm{CE}_0 + \frac{1}{2}(\Sigma L(\phi, \eta) + rI)^{-1}\Sigma L(\phi, \eta)b \cdot b + (\Sigma L(\phi, \eta) + rI)^{-1}b \cdot L(\phi, \eta|a) \ , \quad (36)$$

and the principal's optimal expected surplus is

$$U^p(a) = \frac{1}{2}(I - r(\Sigma L(\phi, \eta) + rI)^{-1})b \cdot b - \mathrm{CE}_0 \ . \quad (37)$$

Proposition 3 *For the standard LEN model with full commitment, the principal strictly prefers* η^2 *to* η^1 *for implementing the optimal action for all benefit vectors* b*, if, and only if,* η^2 *strictly dominates* η^1 *in the VCM sense,* $\Sigma L(\phi, \eta^2) > \Sigma L(\phi, \eta^1)$*. A weak preference relation in all agencies is equivalent to the weak VCM condition.*

Renegotiation

In the renegotiation case, both the agent's second-period action and the agent's compensation contract have to be ex post efficient at renegotiation time; the principal can no longer commit to inducing a given set of actions. The equilibrium in the standard LEN model with renegotiation is the same as that in Gibbons and Murphy (1992), Indjejikian and Nanda (1999), Christensen et al. (2003, 2005), and Şabac (2007, 2008). Let c^I and c^R denote the initial and renegotiated contract offers, respectively. As in the previous section, I proceed by backward induction. I can apply Lemma 3 in Christensen et al. (2010) at renegotiation time to determine the optimal second-period action a_2, the ex post form of the optimal contract and the ex post principal's payoff. The optimal second-period action is

$$a_2 = (\Sigma L(\phi_2, \eta) + rI)^{-1}\Sigma L(\phi_2, \eta)b_2 = (\Sigma L_2(\phi, \eta) + rI)^{-1}\Sigma L_2(\phi, \eta)b_2 \ . \quad (38)$$

The efficient ex post contract offered by the principal, based on his conjecture $\hat{a}_1$, is given by (one can also substitute the optimal action (38) into the efficient contract (16))

$$c^R(y) = \mathrm{CE}_1(c^I(y)|\hat{a}_1, a_2) + \frac{1}{2}(\Sigma L_2(\phi,\eta) + rI)^{-1}\Sigma L_2(\phi,\eta)b_2 \cdot b_2 \\ + (\Sigma L_2(\phi,\eta) + rI)^{-1}b_2 \cdot L_2(\phi,\eta|\hat{a}_1, a_2)\ . \quad (39)$$

Finally, the principal's expected utility at renegotiation time is given by

$$U^p = b_1 \cdot \hat{a}_1 - \mathrm{CE}_1(c^I(y)|\hat{a}_1, a_2) + \frac{1}{2}(I - r(\Sigma L_2(\phi,\eta) + rI)^{-1})b_2 \cdot b_2\ . \quad (40)$$

I solve the agency problem by restricting attention to the linear renegotiation-proof contract. This is without loss of generality: Christensen, Feltham, and Şabac (2003, 2005) and Şabac (2007) demonstrate the renegotiation-proofness principle for LEN models. In what follows, let $c^R(y)$ denote the optimal linear renegotiation-proof contract. Note that $\mathrm{CE}_1(c^I(y)|\hat{a}_1, a_2) + \frac{1}{2}(\Sigma L_2(\phi,\eta) + rI)^{-1}\Sigma L_2(\phi,\eta)b_2 \cdot b_2$ is a function of y_1 only and will be denoted by $c_1^R(y_1)$. Moreover, any contract of the form

$$c(y) = c_1(y_1) + (\Sigma L_2(\phi,\eta) + rI)^{-1}b_2 \cdot L_2(\phi,\eta|\hat{a}_1, a_2) \quad (41)$$

is renegotiation-proof because it is efficient ex post at renegotiation time (that is satisfies the renegotiation-proofness condition (17)) and induces the ex post optimal action (38). The converse is also true because any contract that is renegotiation-proof is efficient ex post and induces the optimal action (38), and therefore satisfies (39) with $c^I = c^R$. Thus, a contract is renegotiation-proof if, and only if, it is of the form (41).

The additive separability obtained in (41) for the renegotiation-proof contract simplifies the analysis in two significant ways. First, to determine the optimal renegotiation-proof contract, it suffices to determine the ex ante optimal $c_1^R(y_1)$. This is a linear contract in y_1, and I can follow a similar procedure to that in the previous section. Second, the two parts of the contract are stochastically independent, see (19). The renegotiation-proofness constraint (41) sets part of the contract and the induced second-period action, so the principal's problem at the initial contract date is to optimize over $c_1(y_1)$ and the induced first-period effort a_1:

$$\max_{c_1(y_1)\in C, a_1} b\cdot a - \mathrm{E}[c(y)|a] \ , \tag{42}$$

subject to the agent's participation constraint (4), the incentive compatibility constraint

$$\nabla_{a_1}\mathrm{CE}(c(y)|a_1,a_2) = \nabla_{a_1}\mathrm{E}(c(y)|a_1,a_2) - \nabla_{a_1}\kappa(a) = 0 \ , \tag{43}$$

and the renegotiation-proofness constraint (41), which also determines the induced second-period action a_2 according to (38).

The agent's certainty equivalent conditional on the anticipated action choice a_1 and on the principal's conjecture $\hat{a}_1$ is

$$\begin{aligned}\mathrm{CE}_0(c(y_1,y_2)|a_1,\hat{a}_1) &= \int c_1(y_1)\bar{\phi}_1(y_1|a_1,\eta)\ dy_1 - \frac{1}{2}a_1\cdot a_1 \\ &+ (\Sigma L_2(\phi,\eta)+rI)^{-1}b_2\cdot \iint L_2(\phi,\eta|\hat{a}_1,a_2)\phi(y_1,y_2|a_1,a_2,\eta)\ dy_1dy_2 \\ &\qquad - \frac{1}{2}r\mathrm{var}(c_1(y_1)) - \frac{1}{2}\Sigma L_2(\phi,\eta)(\Sigma L_2(\phi,\eta)+rI)^{-1}b_2\cdot b_2 \ .\end{aligned} \tag{44}$$

In equilibrium, the agent's action choice a_1 coincides with the principal's conjecture, so that the agent's contract acceptance constraint becomes

$$\begin{aligned}&\int c_1(y_1)\bar{\phi}_1(y_1|a_1,\eta)\ dy_1 - \frac{1}{2}a_1\cdot a_1 - \frac{1}{2}r\mathrm{var}(c_1(y_1)) \\ &\qquad - \frac{1}{2}\Sigma L_2(\phi,\eta)(\Sigma L_2(\phi,\eta)+rI)^{-1}b_2\cdot b_2 \geq \mathrm{CE}_0 \ .\end{aligned} \tag{45}$$

The agent's incentive compatibility constraint follows from first differentiating his certainty equivalent (44) conditional on choosing action a_1 and on the principal's conjecture $\hat{a}_1$, and then setting $a_1 = \hat{a}_1$ (see the details in the derivation of (24))

$$\int c_1(y_1) \nabla_{a_1} \bar{\phi}_1(y_1|a_1, \eta)\, dy_1 \\ + \operatorname{cov}(L_1(\phi, \eta|a_1, a_2), L_2(\phi, \eta|a_1, a_2))(\Sigma L_2(\phi, \eta) + rI)^{-1} b_2 - a_1 = 0 \ . \quad (46)$$

Let $X = (\Sigma L_1(\bar{\phi}_1, \eta) + rI)^{-1}$, $Z = (\Sigma L_2(\phi, \eta) + rI)^{-1}$, and let $B = \operatorname{cov}(L_1(\phi, \eta), L_2(\phi, \eta))$ as before. With this notation, the agent's participation constraint (45) becomes

$$\int c_1(y_1) \bar{\phi}_1(y_1|a_1, \eta)\, dy_1 - \frac{1}{2} r \int (c_1(y_1) - \mathrm{E}[c_1(y_1)])^2 \bar{\phi}_1(y_1|a_1, \eta)\, dy_1 \\ - \frac{1}{2} a_1 \cdot a_1 - \frac{1}{2}(I - rZ) b_2 \cdot b_2 \geq \mathrm{CE}_0 \ , \quad (47)$$

and the agent's incentive compatibility constraint becomes

$$\int c_1(y_1) L_1(\bar{\phi}_1, \eta|a_1) \bar{\phi}_1(y_1|a_1, \eta)\, dy_1 + BZb_2 - a_1 = 0 \ . \quad (48)$$

From now on, I follow the proof of Lemma 3 in Christensen et al. (2010) with the necessary adjustments for the participation and incentive compatibility constraints (47) and (48). This is possible because the renegotiation-proofness constraint (41) separates the contract into two independent parts, $c_1(y_1)$ and $Zb_2 \cdot L_2(\phi, \eta|a_1, a_2)$; the latter enter the participation and incentive compatibility constraints as constants independent of $c_1(y_1)$. Thus, $c_1(y_1)$ can be determined as in the one-period case.

Following the familiar procedure, an efficient contract $c_1(y_1)$ is

$$c_1(y_1) = \mathrm{E}[c_1(y_1)] + \frac{\mu_1}{r} \cdot L_1(\bar{\phi}_1, \eta|a_1) \ , \quad (49)$$

where μ_1 is the multiplier on the agent's incentive compatibility constraint (48). Substituting this efficient contract into the agent's incentive compatibility constraint (48) gives

$$a_1 = BZb_2 + \Sigma L_1(\bar{\phi}_1, \eta)\frac{\mu_1}{r} \ . \tag{50}$$

Substituting back in the principal's expected utility and optimizing over μ_1 yields the following first-order condition,

$$\Sigma L_1(\bar{\phi}_1, \eta)\frac{\mu_1}{r} = \Sigma L_1(\bar{\phi}_1, \eta)X(b_1 - BZb_2) \ , \tag{51}$$

so that the optimal action induced in the first period is

$$a_1 = \Sigma L_1(\bar{\phi}_1, \eta)X(b_1 - BZb_2) + BZb_2 \ . \tag{52}$$

Note that, as in the single-period case, the optimal action is uniquely determined, even if the multiplier μ_1 is not. The same is true for the optimal contract and the principal' expected surplus. To summarize, I have proved the following proposition.

Proposition 4 *For the standard LEN model with renegotiation, the principal's expected surplus is determined in equilibrium by*

$$U^{pr} = \frac{1}{2}\left[I - r(\Sigma ER(\phi, \eta) + rI)^{-1}\right](b_1, b_2) \cdot (b_1, b_2) - \mathrm{CE}_0 \ , \tag{53}$$

where the matrix $(\Sigma ER(\phi, \eta) + rI)^{-1}$ *is given by*

$$(\Sigma ER(\phi, \eta) + rI)^{-1} = \begin{bmatrix} X & -XBZ \\ -ZB^*X & Z + ZB^*XBZ \end{bmatrix} \ . \tag{54}$$

Using again Lemma 5, I can derive a sufficient condition for ranking information systems in the problem with renegotiation.

Corollary 3 *In the standard LEN model with renegotiation, for any marginal benefit vector* $b = (b_1, b_2)$*, the principal strictly prefers* η^2 *to* η^1 *if*

$$\Sigma ER(\phi|\eta^2) > \Sigma ER(\phi|\eta^1) \ , \tag{55}$$

where the inequality is in the usual sense that the difference between the two matrices is positive-definite and the matrix $\Sigma ER(\phi)$ *is given by*

$$\Sigma ER(\phi) + rI = \begin{bmatrix} X^{-1} + BZB^* & B \\ B^* & Z^{-1} \end{bmatrix} = \begin{bmatrix} A^{-1} + BZB^* & B \\ B^* & C^{-1} \end{bmatrix} + rI \ . \tag{56}$$

For a weak preference relation, one replaces the strict inequality in (55) *by a weak one. The ranking is valid even if the matrices* $\Sigma ER(\phi|\eta^i)$ *are singular.*

For the agency with renegotiation, $\Sigma R(\phi)$ corresponds to the variance-covariance matrix of the likelihood function, $\Sigma L(\phi)$, from the agency with full commitment. In particular, note that the variance-covariance matrix of the likelihood function in the full commitment dynamic problem is

$$\Sigma L(\phi, \eta) = \begin{bmatrix} \Sigma L_1(\phi, \eta) & \Sigma L_{12}(\phi, \eta) \\ \Sigma L_{12}(\phi, \eta)^* & \Sigma L_2(\phi, \eta) \end{bmatrix} \ . \tag{57}$$

I now write the matrix $\Sigma R(\phi, \eta)$ in a way that allows a comparison with the variance-covariance of the likelihood function and a comparison between the full commitment case (see Lemma 3, (16) in Christensen et al. 2010) and (31), (32)

$$\Sigma R(\phi, \eta) = \begin{bmatrix} \Sigma L_1(\bar{\phi}_1, \eta) + \Sigma L_{12}(\phi, \eta) \Sigma L_2(\phi, \eta)^{-1} \Sigma L_{12}(\phi, \eta)^* & \Sigma L_{12}(\phi, \eta) \\ \Sigma L_{12}(\phi, \eta)^* & \Sigma L_2(\phi, \eta) \end{bmatrix} \ . \tag{58}$$

Similarly, I write the matrix $\Sigma ER(\phi, \eta)$ in a way that allows a comparison with the variance-covariance of the likelihood function and a comparison between the full commitment case (see Lemma 3, (16) in Christensen et al. 2010) and (53), (54)

$$\Sigma ER(\phi,\eta) = \begin{bmatrix} \Sigma L_1(\bar{\phi}_1,\eta) + \Sigma L_{12}(\phi,\eta)(\Sigma L_2(\phi,\eta) + rI)^{-1}\Sigma L_{12}(\phi,\eta)^* & \Sigma L_{12}(\phi,\eta) \\ \Sigma L_{12}(\phi,\eta)^* & \Sigma L_2(\phi,\eta) \end{bmatrix} . \quad (59)$$

Note that the three matrices are identical, except for the upper left corner: with full commitment we have both for implementing arbitrary actions and on the equilibrium path $\Sigma L_1(\phi,\eta)$; with renegotiation we have $\Sigma L_1(\bar{\phi}_1)+\Sigma L_{12}(\phi,\eta)\Sigma L_2(\phi,\eta)^{-1}\Sigma L_{12}(\phi,\eta)^*$ for implementing arbitrary actions and $\Sigma L_1(\bar{\phi}_1) + \Sigma L_{12}(\phi,\eta)(\Sigma L_2(\phi,\eta) + rI)^{-1}\Sigma L_{12}(\phi,\eta)^*$ on the equilibrium path. If the two periods are stochastically and technologically independent, all three matrices coincide, because $\Sigma L_{12}(\phi,\eta) = 0$ and $\Sigma L_1(\bar{\phi}_1,\eta) = \Sigma L_1(\phi,\eta)$. In other words, if the periods are stochastically and technologically independent, there is no loss due to contract renegotiation, either for implementing exogenously given actions, or on the equilibrium path.

5 Loss from renegotiation

The likelihood ratios and their variance-covariance matrix are given by $L(\phi,\eta|a_1,a_2) = M^*\Sigma^{-1}(y - Ma)$ and $\Sigma L(\phi,\eta) = M^*\Sigma^{-1}M$. With the notation previously introduced, $L(\phi,\eta|a_1,a_2) = (L_1(\phi,\eta|a_1,a_2), L_2(\phi,\eta|a_1,a_2))$ and

$$\Sigma L(\phi,\eta) = \begin{bmatrix} \Sigma L_1(\phi,\eta) & \Sigma L_{12}(\phi,\eta) \\ \Sigma L_{21}(\phi,\eta) & \Sigma L_2(\phi,\eta) \end{bmatrix} , \quad (60)$$

where $\Sigma L_{21}(\phi,\eta) = \Sigma L_{12}(\phi,\eta)^*$. The next lemma gives their explicit characterizations in terms of the sensitivity and variance-covariance of the performance measures.

Lemma 8 *Let* $D = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$ *denote the posterior variance-covariance of* y_2 *conditional on* y_1*. The variance-covariance of the likelihood ratio of the performance measures is given by* $\Sigma L_1(\phi,\eta) = M_{11}^*\Sigma_{11}^{-1}M_{11} + (M_{21}^* - M_{11}^*\Sigma_{11}^{-1}\Sigma_{12})D^{-1}(M_{21} - \Sigma_{21}\Sigma_{11}^{-1}M_{11})$,

$\Sigma L_{12}(\phi,\eta) = (M_{21}^* - M_{11}^*\Sigma_{11}^{-1}\Sigma_{12})D^{-1}M_{22}$, *and* $\Sigma L_2(\phi,\eta) = M_{22}^* D^{-1}M_{22}$. *In addition, the variance-covariance of the likelihood ratio for the marginal distribution of the first-period performance measure is* $\Sigma \bar{L}_1(\phi,\eta) = M_{11}^*\Sigma_{11}^{-1}M_{11}$.

The next result characterizes conditions under which there is no loss from renegotiation if the principal is committed to implementing a given action sequence.

Proposition 5 *Assume that* M_{tt} *has full column rank* $m_t \leq n_t$, $t = 1, 2$ *and that the marginal cost of effort for all implementable actions spans the entire space* $\mathbb{R}^{m_1} \times \mathbb{R}^{m_2}$. *Then, there is no loss from renegotiation when implementing any actions* (a_1, a_2) *if, and only if,*

$$\Sigma L_1(\phi,\eta) = \Sigma\bar{L}_1(\phi,\eta) + \Sigma L_{12}(\phi,\eta)\Sigma L_2(\phi,\eta)^{-1}\Sigma L_{12}(\phi,\eta)^* \ . \tag{61}$$

In particular, there is no loss from renegotiation in implementing any actions (a_1, a_2), *if one of the following sufficient conditions holds:*

(i)There are equal numbers of tasks and performance measures in the second period, $m_2 = n_2$*;*

(ii) There are more performance measures than tasks in the second period, $m_2 < n_2$, *and*

$$M_{21} - \Sigma_{12}^*\Sigma_{11}^{-1}M_{11} = 0 \ ; \tag{62}$$

(iii) There are more performance measures than tasks in the second period, $m_2 < n_2$, *and*

$$D = M_{22}(M_{22}^*D^{-1}M_{22})^{-1}M_{22}^* \tag{63}$$

where $D = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$ *is the posterior variance-covariance of* y_2 *conditional on* y_1.

The reason there is no loss from renegotiation in the cases covered by condition (i) in Proposition 5 is that the second-period incentive compatibility constraints alone determine the second-period incentives and there is no scope for a more efficient ex post contract that induces the same second-period action. The fact that there is no loss from renegotiation if the principal can commit to the second period incentive rates has been noted by Christensen et al. (2003) in a particular case. Their setting corresponds to condition (i) in Proposition 5 with $m_2 = n_2 = 1$ for implementing the actions that are optimal under full commitment.

Condition (ii) in Proposition 5 states that, if all the performance information pertaining to a_1 is timely, then there is no loss to renegotiation. Condition (ii) is also equivalent to the second-period performance measures not being conditionally controllable with respect to a_1 given y_1. If the second-period performance information, conditional on first-period information, is not informative with respect to a_1, all the performance information with respect to a_1 is timely, that is available prior to renegotiation. In this case, there is no loss from renegotiation in providing incentives for a_1 and there is no loss from contract renegotiation in implementing given actions a_1, a_2. The argument above is also closely related to the conditional controllability principle described by Antle and Demski (1988). If y_2 is not conditionally controllable with respect to a_1 then it has no value in providing incentives for a_1, even under full commitment. Because renegotiation (when implementing exogenous actions) only prevents the principal from using y_2 to control a_1, when y_2 is not conditionally controllable, there will be no loss from renegotiation.

Lemma 9 *Let $\phi(y|a,\eta) = \bar{\phi}_1(y_1|a_1,\eta)\phi_2(y|a,\eta)$, where $\bar{\phi}_1$ is the marginal distribution of y_1 and ϕ_2 is the distribution of y_2 conditional on y_1, a_1. Then the following identities hold*

$$\Sigma L_1(\phi,\eta) = \Sigma L_1(\bar{\phi}_1,\eta) + \Sigma L_1(\phi_2,\eta)\,, \tag{64}$$

$$\Sigma L_{12}(\phi,\eta) = \operatorname{cov}(L_1(\phi_2|a,\eta), L_2(\phi|a,\eta)) = \operatorname{cov}(L_1(\phi_2|a,\eta), L_2(\phi_2|a,\eta))\ . \tag{65}$$

Using Lemma 9 I can further simplify the no loss from renegotiation condition in Proposition 5.

Corollary 4 *Assume that M_{tt} has full column rank $m_t \leq n_t$, $t = 1, 2$ and that the marginal cost of effort for all implementable actions spans the entire space $\mathbb{R}^{m_1} \times \mathbb{R}^{m_2}$. Then, there is* *no loss from renegotiation when implementing any actions* $a = (a_1, a_2)$ *if, and only if,*

$$\Sigma L_1(\phi_2, \eta) = \Sigma L_{12}(\phi_2, \eta)\Sigma L_2(\phi_2, \eta)^{-1}\Sigma L_{12}(\phi_2, \eta)^* \; . \tag{66}$$

In particular, if $L_1(\phi_2|a, \eta) = 0$ *for all* $a = (a_1, a_2)$*, there is no loss from renegotiation in implementing any actions* $a = (a_1, a_2)$*.*

The left hand side in condition (66) represents the contribution of second-period performance information in controlling a_1 under full commitment, see the decomposition (64) of $\Sigma L_1(\phi, \eta)$. The right hand side of (66) represents the contribution of second-period performance information in controlling a_1 under renegotiation, see the upper left corner in (35) and (65). Under renegotiation, the second-period performance measures are only used indirectly and provide first-period incentives in the form of an externality as long as the likelihood ratios $L_1(\phi|a, \eta)$ and $L_2(\phi|a, \eta)$ are correlated, see (65) and (28) in Lemma 3.

The conditional controllability of second-period performance information with respect to the first-period action reappears in a slightly different form: y_2 is not conditionally controllable with respect to a_1 given y_1 if, and only if, $L_1(\phi_2|a, \eta) = 0$. If $L_1(\phi_2|a, \eta) = 0$, then both sides in (66) are zero. This can be easily seen because, being a mean-zero normally distributed random variable, $L_1(\phi_2|a, \eta) = 0$ if, and only if $\Sigma L_1(\phi_2, \eta) = 0$. In addition, $\Sigma L_1(\phi_2, \eta) = (M_{21}^* - M_{11}^*\Sigma_{11}^{-1}\Sigma_{12})D^{-1}(M_{21} - \Sigma_{12}^*\Sigma_{11}^{-1}M_{11})$; thus, $\Sigma L_1(\phi_2, \eta) = 0$, if and only if, $M_{21} - \Sigma_{12}^*\Sigma_{11}^{-1}M_{11} = 0$. In this case, the principal can use only the first-period performance to control a_1 both under full commitment and under renegotiation.

A similar result to Corollary 4 obtains on the equilibrium path in the standard LEN model. By Lemma 7 and Proposition 4, there is no loss from renegotiation on the equilibrium path for all marginal benefit vectors $b = (b_1, b_2)$, if, and only if $\Sigma L(\phi, \eta) = \Sigma ER(\phi, \eta)$. Using Lemma 9 and the explicit characterization of $\Sigma ER(\phi, \eta)$ in (59), I obtain the following result.

Corollary 5 *There is no loss from renegotiation on the equilibrium path for any marginal* *benefit vectors $b = (b_1, b_2)$ if, and only if,*

$$\Sigma L_1(\phi_2, \eta) = \Sigma L_{12}(\phi_2, \eta) \left(\Sigma L_2(\phi_2, \eta) + rI\right)^{-1} \Sigma L_{12}(\phi_2, \eta)^* \ . \tag{67}$$

In particular, if $L_1(\phi_2 | a, \eta) = 0$ for all $a = (a_1, a_2)$, there is no loss from renegotiation on the equilibrium path.

It follows that the conditional controllability of y_2 with respect to a_1 is necessary for the principal to incur an expected loss from renegotiation, both for exogenously specified actions, and on the equilibrium path. However, the conditional controllability of y_2 with respect to a_1 is not sufficient for losses from renegotiation to occur. In particular, it is possible for the principal to have no losses from renegotiation when implementing exogenous actions and, at the same time to have losses from renegotiation on the equilibrium path. To see this, assume that (66) is satisfied with the additional requirement that y_2 is conditionally controllable with respect to a_1 and

$$\Sigma L_1(\phi_2, \eta) = \Sigma L_{12}(\phi_2, \eta) \Sigma L_2(\phi_2, \eta)^{-1} \Sigma L_{12}(\phi_2, \eta)^* \neq 0 \ . \tag{68}$$

Then, by Corollary 4, there is no loss from renegotiation in implementing exogenous actions $a = (a_1, a_2)$. However, $\Sigma L_2(\phi_2, \eta) < \Sigma L_2(\phi_2, \eta) + rI$ implies, by Lemma 5, that $\Sigma L_2(\phi_2, \eta)^{-1} > (\Sigma L_2(\phi_2, \eta) + rI)^{-1}$. Consequently,

$$\begin{aligned} \Sigma L_1(\phi_2, \eta) &= \Sigma L_{12}(\phi_2, \eta) \Sigma L_2(\phi_2, \eta)^{-1} \Sigma L_{12}(\phi_2, \eta)^* \\ &> \Sigma L_{12}(\phi_2, \eta) (\Sigma L_2(\phi_2, \eta) + rI)^{-1} \Sigma L_{12}(\phi_2, \eta)^* \ , \end{aligned} \tag{69}$$

and (67) in Corollary 5 is not satisfied; thus, there are losses from renegotiation on the equilibrium path for most benefit vectors $b = (b_1, b_2)$. Such a case is further illustrated in Example 1 below.

Examples

1. One task in each period, one long-term task.

$$\begin{cases} y_{11} = m_{11}a_1 + \varepsilon_1 \\ y_{21} = m_{21}a_1 + m_{22}a_2 + \varepsilon_2 \end{cases} \tag{70}$$

The sensitivity and variance-covariance matrices of the signals are given by

$$M = \begin{bmatrix} m_{11} & 0 \\ m_{21} & m_{22} \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} . \tag{71}$$

The variances and covariances of the likelihood ratios are $\Sigma L(\bar{\phi}_1, \eta) = m_{11}^2/\sigma_1^2$, and

$$\begin{aligned} \Sigma L_1(\phi, \eta) &= \frac{m_{11}^2}{(1-\rho^2)\sigma_1^2} - 2\rho\frac{m_{11}m_{21}}{(1-\rho^2)\sigma_1\sigma_2} + \frac{m_{21}^2}{(1-\rho^2)\sigma_2^2} , \\ \operatorname{cov}(L_1(\phi, \eta|a_1), L_2(\phi, \eta|a_2)) &= \frac{m_{22}}{(1-\rho^2)\sigma_2^2}\left[m_{21} - \rho\frac{\sigma_2}{\sigma_1}m_{11}\right] , \\ \Sigma L_2(\phi, \eta) &= \frac{m_{22}^2}{(1-\rho^2)\sigma_2^2} . \end{aligned} \tag{72}$$

For $m_{21} = 0$, this is the model with one task in each period and correlated performance measures in Indjejikian and Nanda (1999) and Christensen et al. (2003, 2005). Otherwise, it is a two-period version of the model considered in Şabac (2008), with one long-term task in each period.

In this case, there is no loss from renegotiation in implementing a chosen sequence of actions. The reason is that the incentive compatibility constraints determine the contract and renegotiation has no impact. This is no longer true on the equilibrium path because the principal's inability to commit to a second-period action leads to ex ante inefficient actions and a loss from renegotiation.[5]

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[5] Christensen et al. (2003), p. 430 identify the source of the expected loss from renegotiation as "the inability of the principal to commit ex-ante to the second-period incentive rate." In their model, commitment to the second-period incentive rate is the same as commitment to a second-period action. However, the above analysis identifies the source of the loss from renegotiation as the inability to commit to the second-period

2. One task in each period, one long-term task.

$$\begin{cases} y_{11} = m_{11}a_1 + \varepsilon_{11} \\ y_{21} = m_{21}a_1 + \varepsilon_{21} \quad y_{22} = m_{22}a_2 + \varepsilon_{22} \end{cases} \tag{73}$$

The sensitivity and variance-covariance matrices of the signals are given by

$$M = \begin{bmatrix} m_{11} & 0 \\ m_{21} & 0 \\ 0 & m_{22} \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \sigma_{11}^2 & 0 & 0 \\ 0 & \sigma_{21}^2 & 0 \\ 0 & 0 & \sigma_{22}^2 \end{bmatrix} . \tag{74}$$

The variances and covariances of the likelihood ratios are $\Sigma L(\bar{\phi}_1, \eta) = m_{11}^2/\sigma_{11}^2$, and

$$\Sigma L_1(\phi, \eta) = \frac{m_{11}^2}{\sigma_{11}^2} + \frac{m_{21}^2}{\sigma_{21}^2} \ , \quad \Sigma L_{12}(\phi, \eta) = 0 \ , \quad \Sigma L_2(\phi, \eta) = \frac{m_{22}^2}{\sigma_{22}^2} \ . \tag{75}$$

In this case there is loss from renegotiation in implementing arbitrary actions, if, and only if, $m_{21} \neq 0$. The reason is that renegotiation effectively eliminates y_{21} from the final contract and increases the cost of inducing a_1.

3. One task in each period, additional non-controllable performance information.

$$\begin{cases} y_{11} = m_{11}a_1 + \varepsilon_{11} \\ y_{21} = \varepsilon_{21} \qquad\qquad\quad y_{22} = m_{22}a_2 + \varepsilon_{22} \end{cases} \tag{76}$$

The sensitivity and variance-covariance matrices of the signals are given by

$$M = \begin{bmatrix} m_{11} & 0 \\ 0 & 0 \\ 0 & m_{22} \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \sigma_{11}^2 & \rho_1\sigma_{11}\sigma_{21} & 0 \\ \rho_1\sigma_{11}\sigma_{21} & \sigma_{21}^2 & \rho_2\sigma_{21}\sigma_{22} \\ 0 & \rho_2\sigma_{21}\sigma_{22} & \sigma_{22}^2 \end{bmatrix} . \tag{77}$$

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action. Thus, in models such as that in Christensen et al. (2003), lack of commitment to actions creates a loss of efficiency on the equilibrium path, while the renegotiation of contracts in the presence of commitment to actions does not lead to any losses. These models are covered by condition (i) in Proposition 5.

The variances and covariances of the likelihood ratios are $\Sigma L(\bar{\phi}_1, \eta) = m_{11}^2/\sigma_{11}^2$, and

$$\Sigma L_1(\phi, \eta) = \frac{m_{11}^2}{\sigma_{11}^2} \frac{1-\rho_2^2}{1-\rho_1^2-\rho_2^2} \;, \Sigma L_{12}(\phi, \eta) = 0 \;, \Sigma L_2(\phi, \eta) = \frac{m_{22}^2}{\sigma_{22}^2} \frac{1-\rho_1^2}{1-\rho_1^2-\rho_2^2} \;. \tag{78}$$

A direct calculation shows that

$$\Sigma L_1(\phi, \eta) - \left(\Sigma \bar{L}_1(\phi, \eta) + \Sigma L_{12}(\phi, \eta) \Sigma L_2(\phi, \eta)^{-1} \Sigma L_{12}(\phi, \eta)^*\right) = \frac{m_{11}^2}{\sigma_{11}^2} \frac{\rho_1^2}{1-\rho_1^2} \;. \tag{79}$$

Thus, by proposition 5, there is loss from renegotiation in implementing arbitrary actions, if, and only if, $\rho_1 = 0$. The intuition is simple. Assume $\rho_1 \neq 0$. Then, ε_{21} can be used under full commitment, but not under renegotiation, to "filter" the noise in the first-period performance. This is independent of whether the same information is used to filter the noise in the second-period performance measure (this only happens when $\rho_2 \neq 0$).

Note that the dynamic conditional controllability of y_2 with respect to a_1 is only necessary, but not sufficient for the principal to incur a loss from renegotiation. Example 1 above illustrates the case when y_2 is dynamically conditionally controllable with respect to a_1 whenever

$$M_{21} - \Sigma_{21} \Sigma_{11}^{-1} M_{11} = m_{21} - \rho \frac{\sigma_2}{\sigma_1} m_{11} \neq 0 \;. \tag{80}$$

Thus, there is no loss from renegotiation even when y_2 is conditionally controllable with respect to a_1.

By contrast, in examples 2 and 3, there is no loss from renegotiation, if, and only if, y_2 is not conditionally controllable with respect to a_1. To see this, note that in examples 2, and 3,

$$M_{21} - \Sigma_{21} \Sigma_{11}^{-1} M_{11} = \begin{bmatrix} m_{21} - \rho_1 \frac{\sigma_{21}}{\sigma_{11}} m_{11} \\ 0 \end{bmatrix} \;, \tag{81}$$

where $\rho_1 = 0$ in example 2 and $m_{21} = 0$ in example 3.

Renegotiation and implementable actions

The key difference between the commitment and renegotiation cases is in the controllability of the first-period action. Renegotiation can result in a loss of controllability of the first-period actions and a restriction of the space of implementable first-period actions. With full commitment, the principal can use the information in both performance measures to control the agent's action a_1. With renegotiation, the principal cannot use the second-period performance measure to control the first-period action because, when the contract is renegotiated, the action is taken and the second performance measure has not been reported; any risky incentive compensation related to the sunk effort is ex post removed as being inefficient.

To further clarify this important point, consider the following stark example with noiseless performance measures in the standard LEN model with renegotiation: $y_1 = m_{11}a_{11} + m_{12}a_{12}$ and $y_2 = m_{21}a_{11} + m_{22}a_{22}$, where $a_1 = (a_{11}, a_{12})$ is a two-task action the agent takes in the first period. If the sensitivity matrix $M = [m_{ij}]$ is of full rank, then any action is implementable and first-best can be attained with full commitment. However, with renegotiation, only actions that satisfy $a_1/a_2 = m_{11}/m_{12}$ can be implemented. The key is that renegotiation has effectively removed y_2 from the principal's arsenal in controlling the two tasks. Note that this is very much a multi-task version of Fudenberg and Tirole (1990); in their case, with a single task and a single performance measure, the space of implementable actions is reduced to the null action.

To better understand the restrictions on the actions that can be implemented, compare the characterization of an implementable action in the first period in terms of the corresponding multiplier μ_1. In the full commitment case, from Lemma 1, (12) in Christensen et al. (2010),

$$a_1 = \Sigma L_1(\phi, \eta)\frac{\mu_1}{r} + \text{cov}(L_1(\phi, \eta|a_1), L_2(\phi, \eta|a_2))\frac{\mu_2}{r} \ . \tag{82}$$

In the renegotiation case, from (28),

$$a_1 = \Sigma L_1(\bar{\phi}_1, \eta)\frac{\mu_1}{r} + +\text{cov}(L_1(\phi, \eta|a_1), L_2(\phi, \eta|a_2))\frac{\mu_2}{r} \ . \tag{83}$$

The key difference between the space of implementable actions a_1 is now apparent: with full commitment it has the same dimension as the image of the variance-covariance of $L_1(\phi, \eta|a_1)$, while with renegotiation it has the same dimension as the image of the variance-covariance of $L_1(\bar{\phi}_1, \eta|a_1)$.[6] In the first case, the principal relies on both signals (the full density function of their joint distribution), while in the second case, the principal relies only on the first signal (the marginal density of the first signal).

6 The value of additional information

Let y, z be joint normally distributed, with mean linearly dependent on the agent's actions $a = (a_1, a_2)$, and variance independent of a. Let $\phi(y, z|a)$ denote the joint density of the distribution of y and z and let $\phi(y|a)$ denote the marginal density of the distribution of y. As before, $\bar{\phi}_1(y_1, z_1|a_1)$, $\bar{\phi}_1(y_1|a_1)$ denote the marginal densities of first-period performance measures; $\phi_2(y, z|a)$, $\phi_2(y|a)$ denote the conditional densities of second-period performance measures.

Definition 1 *Renegotiation sufficient statistic. The variables (y, z) are a renegotiation suf-*

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[6] For a given second-period action, the second-term in either of the two expressions is a constant and does not affect the dimensionality or the "direction" of the linear subspace of implementable actions. For a given benefit vector, these terms also matter, because, where this hyperplane is positioned also matters.

ficient statistic for y *if there exist functions* $h(y, z|a_1)$ *and* $\bar{h}_1(y_1, z_1)$ *such that*

$$\phi(y, z|a) = h(y, z|a_1)\phi(y|a) \tag{84}$$
$$\int h(y, z|a_1)\ dz_2 = \bar{h}_1(y_1, z_1)\ . \tag{85}$$

Note that, given $\phi(y, z|a)$ and $\phi(y|a)$, (84) and (85) always determine two functions $h(y, z|a)$ and $\bar{h}_1(y_1, z_1|a)$; the key requirements are that h does not depend on a_2 and $\bar{h}_1$ does not depend on either a_1 or y_2.

The renegotiation sufficient statistic condition neither implies nor is it implied by the sufficient statistic condition. Recall that the variables (y, z) are a sufficient statistic for y for all a if

$$\phi(y, z|a) = h(y, z)\phi(y|a)\ . \tag{86}$$

The sufficient statistic condition implies (84), but if $\int h(y, z)\ dz_2$ depends on y_2, (85), and thus the renegotiation sufficient statistic condition, are not satisfied. Conversely, if the renegotiation sufficient statistic condition is satisfied, $h(y, z|a_1)$ may depend on a_1 in which case the sufficient statistic condition (86) does not hold.

The sufficient statistic condition (86) is necessary and sufficient for the additional performance measures z to have no incremental value over y in a single-period multi-task model, see Proposition 6 in Christensen et al. (2010). For the dynamic model in this paper, the sufficient statistic condition is necessary and sufficient for the additional information to have no value *under full commitment.* A similar result can be obtained for the renegotiation sufficient statistic condition.

Proposition 6 *Under the LEN model assumptions with renegotiation, assume the marginal effort cost vectors for all actions that are implementable under both information systems* y *and* (y, z) *span the entire space* $\mathbb{R}^{m_1} \times \mathbb{R}^{m_2}$. *Then, the additional performance measures* z *have no incremental value over* y *in implementing any given actions* $a = (a_1, a_2)$, *if and only* *if (y, z) is a renegotiation sufficient statistic for y.*

Note that the renegotiation sufficient statistic result in Proposition 6 holds both for implementing arbitrary given actions (provided the marginal effort cost for all implementable actions spans $\mathbb{R}^{m_1} \times \mathbb{R}^{m_2}$) and on the equilibrium path for all marginal benefit vectors $b = (b_1, b_2)$. The reason is that $\Sigma ER(\phi, y, z) = \Sigma ER(\phi, y)$, if, and only if $\Sigma R(\phi, y, z) = \Sigma R(\phi, y)$.

Corollary 6 *Under the LEN model assumptions with renegotiation, the additional performance measures z have no incremental value over y on the equilibrium path, if and only if (y, z) is a renegotiation sufficient statistic for y.*

Examples

To further illustrate the differences between the two sufficient statistic conditions, consider the following examples.

1. Renegotiation sufficient statistic, but not sufficient statistic.

$$\begin{cases} y_1 = m_1 a_1 + \varepsilon_1 \quad z_1 = \varepsilon_{11} \\ y_2 = m_2 a_2 + \varepsilon_2 \quad z_2 = m_{21} a_1 + \varepsilon_{21} \end{cases} \tag{87}$$

The noise terms are joint normally distributed, have mean zero, and only ε_{11} and ε_{21} are correlated; all others are uncorrelated. In this case, the additional performance information is valuable under full commitment, but has no value under renegotiation. The intuition is simple: renegotiation eliminates z_2 from the effectively useable information and z_1 would be potentially useful only in the presence of z_2, being noise uncorrelated with the other performance information.

2. Sufficient statistic, but not renegotiation sufficient statistic.

$$\begin{cases} y_1 = m_1 a_1 + \varepsilon_1 & & z_1 = m_{21} a_1 + \varepsilon_{21} \\ y_{22} = m_2 a_2 + \varepsilon_2 & y_{21} = m_{21} a_1 + \varepsilon_{21} & z_2 = \emptyset \end{cases} \tag{88}$$

The noise terms are joint normally distributed, have mean zero, and are uncorrelated. In this case, the additional performance information is valuable under renegotiation, but has no value under full commitment. The intuition is again simple: renegotiation eliminates y_{21} from the effectively useable information and makes the otherwise redundant z_1 valuable.

3. Sufficient statistic and renegotiation sufficient statistic.

$$\begin{cases} y_1 = m_1 a_1 + \varepsilon_1 & & z_1 = \emptyset \\ y_{22} = m_2 a_2 + \varepsilon_2 & y_{21} = m_{21} a_1 + \varepsilon_{21} & z_2 = m_{21} a_1 + \varepsilon_{21} \end{cases} \tag{89}$$

The noise terms are joint normally distributed, have mean zero, and are uncorrelated. In this case, the additional information has no value under either full commitment or renegotiation. With full commitment, z_2 is simply redundant; with renegotiation, z_2 is eliminated at the same time as y_{21} so it cannot act as a substitute as in the preceding example.

The key feature of all three examples is that performance information that only pertains to the first-period action cannot be used if it is reported in the second period and the contracts are renegotiated. This introduces a stringent requirement on the timeliness of performance measures under renegotiation that is not present with full commitment. Thus, in the first example, z_2 is not valuable under renegotiation because it is not timely, but would be valuable under full commitment. In the second example, z_1 has value only because it is timely under renegotiation, but would be redundant under full commitment.

Appendix A: Stochastically independent sufficient performance measures

Some of the analysis is easier to follow if all the intertemporal dependencies are reduced to "long-term actions." It is without loss of generality to assume that the two periods are stochastically independent, but technologically interdependent, because the original performance measures can be replaced by the equivalent stochastically independent sufficient performance statistics, see Christensen and Feltham (2005). Denote the density function of the joint distribution of the stochastically independent signals by $\psi(y_1, y_2|a_1, a_2, \eta) = \psi_1(y_1|a_1, \eta)\psi_2(y_2|a_1, a_2, \eta)$. It follows that $\bar{\psi}_1(y_1|a_1, \eta) = \psi_1(y_1|a_1, \eta)$, and the likelihood ratios are $L_1(\psi, \eta|a_1) = L_1(\psi_1, \eta|a_1) + L_1(\psi_2, \eta|a_1)$, and $L_2(\psi, \eta|a_2) = L_2(\psi_2, \eta|a_2)$. In addition, the variance-covariance matrices of the likelihood ratios are $\Sigma L_1(\psi, \eta) = \Sigma L_1(\psi_1, \eta) + \Sigma L_1(\psi_2, \eta)$ and $\mathrm{cov}(L_1(\psi, \eta|a_1), L_2(\psi, \eta|a_2)) = \mathrm{cov}(L_1(\psi_1, \eta|a_1), L_1(\psi_2, \eta|a_1))$.

Let Σ_1 and Σ_2 be the variance-covariance matrices of the two independent performance statistics. The likelihood ratios are determined explicitly as follows. First, for the likelihood ratio with respect to a_1, $L_1(\psi, \eta|a_1) = L_1(\psi_1, \eta|a_1) + L_1(\psi_2, \eta|a_1)$, where

$$\begin{aligned} L_1(\psi_1, \eta|a_1) &= M_{11}^* \Sigma_1^{-1}(y_1 - M_{11}a_1) \ , \\ L_1(\psi_2, \eta|a_1) &= M_{21}^* \Sigma_2^{-1}(y_2 - M_{21}a_1 - M_{22}a_2) \ . \end{aligned} \tag{A1}$$

Then, for the likelihood ratio with respect to a_2, $L_2(\psi, \eta|a_2) = L_2(\psi_2, \eta|a_2)$, where

$$L_2(\psi_2, \eta|a_2) = M_{22}^* \Sigma_2^{-1}(y_2 - M_{21}a_1 - M_{22}a_2) \ . \tag{A2}$$

Using the fact that $\mathrm{cov}(L_1(\psi,\eta|a_1), L_2(\psi,\eta|a_2)) = \mathrm{cov}(L_1(\psi_2,\eta|a_1), L_2(\psi_2,\eta|a_2))$, I have

$$\begin{aligned}
\Sigma L_1(\psi,\eta) &= M_{11}^* \Sigma_1^{-1} M_{11} + M_{21}^* \Sigma_2^{-1} M_{21}\ , \\
\mathrm{cov}(L_1(\psi,\eta|a_1), L_2(\psi,\eta|a_2)) &= M_{21}^* \Sigma_2^{-1} M_{22}\ , \\
\Sigma L_2(\psi,\eta) &= M_{22}^* \Sigma_2^{-1} M_{22}\ .
\end{aligned} \tag{A3}$$

Appendix B: Proofs

Proof of Lemma 4. For an arbitrary partitioned vector $v = (v_1, v_2)$, $Qv \cdot v = Q_{11}v_1 \cdot v_1 + 2v_1 \cdot Q_{12}v_2 + Q_{22}v_2 \cdot v_2$.

If $Q > 0$, then $Qv \cdot v > 0$ for $v = (v_1, 0)$ and $v = (0, v_2)$ imply $Q_{11} > 0$ and $Q_{22} > 0$. Because $Q_{11} > 0$, Q_{11} is invertible and the following identity holds, $Q_{11}(v_1 + Q_{11}^{-1}Q_{12}v_2) \cdot (v_1 + Q_{11}^{-1}Q_{12}v_2) = Q_{11}v_1 \cdot v_1 + 2v_1 \cdot Q_{12}v_2 + Q_{12}^{*}Q_{11}^{-1}Q_{12}v_2 \cdot v_2$. It follows that

$$Qv \cdot v = Q_{11}(v_1 + Q_{11}^{-1}Q_{12}v_2) \cdot (v_1 + Q_{11}^{-1}Q_{12}v_2) + (Q_{22} - Q_{12}^{*}Q_{11}^{-1}Q_{12})v_2 \cdot v_2 \ . \tag{B1}$$

Thus, by setting $v_1 = -Q_{11}^{-1}Q_{12}v_2$ for an arbitrary v_2, $Q > 0$ implies $Q_{22} - Q_{12}^{*}Q_{11}^{-1}Q_{12} > 0$.

For the converse, note that $Q_{11} > 0$ implies the same decomposition (B1) of the quadratic form $Qv \cdot v$ holds. To conclude, $Q_{11} > 0$ and $Q_{22} - Q_{12}^{*}Q_{11}^{-1}Q_{12} > 0$ imply $Qv \cdot v > 0$ for all $v \neq 0$.

For the rest of the proof, a straightforward calculation shows that Q^{-1} is indeed the inverse of Q. □

Proof of Lemma 6 First, I show that $\Sigma\bar{L}_1(\phi, \eta)$ and $\Sigma L_2(\phi, \eta)$ are invertible. By Lemma 8, $\Sigma\bar{L}_1(\phi, \eta) = M_{11}^{*}\Sigma_{11}^{-1}M_{11}$ and $\Sigma L_2(\phi, \eta) = M_{22}^{*}D^{-1}M_{22}$, and these imply that $\Sigma\bar{L}_1(\phi, \eta) \geq 0$ and $\Sigma L_2(\phi, \eta) \geq 0$. For any matrix of the form $M^{*}\Sigma^{-1}M$, it holds that $M^{*}\Sigma^{-1}M = Q^{*}Q$ with $Q = \Sigma^{-1/2}M$. Because $\mathrm{Im}(Q^{*}Q) \subseteq \mathrm{Im}(Q^{*})$ and because $\mathrm{rank}(Q^{*}Q) = \mathrm{rank}(Q^{*})$, it follows that $\mathrm{Im}(Q^{*}Q) = \mathrm{Im}(Q^{*})$. Moreover, because the matrix Σ has full rank $n_t \geq m_t$ in both cases, it follows that $\mathrm{Im}(Q^{*}) = \mathrm{Im}(M^{*}\Sigma^{-1/2}) = \mathrm{Im}(M^{*})$. To conclude, $\mathrm{Im}(M^{*}\Sigma^{-1}M) = \mathrm{Im}(Q^{*}Q) = \mathrm{Im}(Q^{*}) = \mathrm{Im}(M^{*})$ implies that in each case, the corresponding $m_t \times m_t$ matrices have full rank m_t. Thus, both matrices are non-degenerate and positive definite. Applying Lemma 4 to the matrix $R(\phi, \eta)$ proves the rest,

$$\Sigma R(\phi, \eta) = R(\phi, \eta)^{-1} = \begin{bmatrix} A^{-1} + BCB^{*} & B \\ B^{*} & C^{-1} \end{bmatrix} . \ \square \tag{B2}$$

Proof of Lemma 8. If the variance-covariance matrix of the noise terms is non-degenerate, $\Sigma > 0$, then by Lemma 4, $D > 0$ and the joint precision matrix of the performance measures is

$$\Sigma^{-1} = \begin{bmatrix} \Sigma_{11}^{-1}(I + \Sigma_{12}D^{-1}\Sigma_{21}\Sigma_{11}^{-1}) & -\Sigma_{11}^{-1}\Sigma_{12}D^{-1} \\ -D^{-1}\Sigma_{21}\Sigma_{11}^{-1} & D^{-1} \end{bmatrix} . \tag{B3}$$

The rest of the proof is a straightforward calculation using the precision matrix characterized above and collecting terms. □

Proof of Proposition 5. From Lemma 1 and Proposition 2, there is no loss from renegotiation if, and only if $\Sigma L(\phi, \eta)^{-1} = R(\phi, \eta)$. The condition is sufficient in general and necessary if the marginal cost of effort for all implementable actions spans the entire space $\mathbb{R}^{m_1} \times \mathbb{R}^{m_2}$. By Lemma 6, $R(\phi, \eta)$ is invertible and all actions are implementable. Thus, there is no loss from renegotiation, if and only if $\Sigma L(\phi, \eta) = R(\phi, \eta)^{-1}$, provided $\nabla_{a_t}\kappa_t(a_t)$ spans $\mathbb{R}^{m_t}$. Using the characterization of $R(\phi, \eta)^{-1}$ from Lemma 6 proves (61). Using the explicit expressions for $\Sigma L(\phi, \eta)$ and $\Sigma \bar{L}_1(\phi, \eta)$from Lemma 8 gives the following identity:

$$\begin{aligned} &\Sigma L_1(\phi, \eta) - \Sigma \bar{L}_1(\phi, \eta) - \Sigma L_{12}(\phi, \eta)\Sigma L_2(\phi, \eta)^{-1}\Sigma L_{12}(\phi, \eta)^* \\ &\quad = (M_{21}^* - M_{11}^*\Sigma_{11}^{-1}\Sigma_{12})D^{-1}\left[D - M_{22}(M_{22}^*D^{-1}M_{22})^{-1}M_{22}^*\right]D^{-1}(M_{21} - \Sigma_{12}^*\Sigma_{11}^{-1}M_{11}) \end{aligned} \tag{B4}$$

The above is zero whenever conditions (62) or (63) hold. This proves the sufficiency of (ii) and (iii) in the proposition. To prove (i), note that (63) holds whenever M_{22} is invertible; but $m_2 = n_2$ and $\text{rank}(M_{22}) = m_2$ implies M_{22} is invertible. □

Proof of Lemma 9. From the definitions of the marginal and conditional distributions it follows that

$$\begin{aligned}L_1(\phi|a,\eta) &= \frac{\nabla_{a_1}\phi(y|a,\eta)}{\phi(y|a,\eta)} = \frac{\nabla_{a_1}\left[\bar{\phi}_1(y_1|a_1,\eta)\phi_2(y|a,\eta)\right]}{\bar{\phi}_1(y_1|a_1,\eta)\phi_2(y|a,\eta)} \\ &= \frac{\left[\nabla_{a_1}\bar{\phi}_1(y_1|a_1,\eta)\right]\phi_2(y|a,\eta)}{\bar{\phi}_1(y_1|a_1,\eta)\phi_2(y|a,\eta)} + \frac{\bar{\phi}_1(y_1|a_1,\eta)\nabla_{a_1}\left[\phi_2(y|a,\eta)\right]}{\bar{\phi}_1(y_1|a_1,\eta)\phi_2(y|a,\eta)} \\ &= L_1(\bar{\phi}_1|a_1,\eta) + L_1(\phi_2|a,\eta)\ .\end{aligned} \tag{B5}$$

Furthermore, the decomposition in (B5) is orthogonal in that

$$\operatorname{cov}(L_1(\bar{\phi}_1|a_1,\eta), L_1(\phi_2|a,\eta)) = 0\ . \tag{B6}$$

To see this is a straightforward calculation as follows,

$$\begin{aligned}\operatorname{cov}(l_{1i}(\bar{\phi}_1), l_{1j}(\phi_2)) &= \iint l_{1i}(\bar{\phi}_1) l_{1j}(\phi_2)\phi\ dy_1 dy_2 = \iint \partial_{a_{1i}}\bar{\phi}_1\ \partial_{a_{1j}}\phi_2\ dy_1 dy_2 \\ &= \int \partial_{a_{1i}}\bar{\phi}_1 \left(\int \partial_{a_{1j}}\phi_2\ dy_2\right)\ dy_1 = \int \partial_{a_{1i}}\bar{\phi}_1 \left(\partial_{a_{1j}} \int \phi_2\ dy_2\right)\ dy_1 = 0\ .\end{aligned} \tag{B7}$$

Similarly,

$$\operatorname{cov}(L_1(\bar{\phi}_1|a_1,\eta), L_2(\phi|a,\eta)) = \operatorname{cov}(L_1(\bar{\phi}_1|a_1,\eta), L_2(\phi_2|a,\eta)) = 0\ . \tag{B8}$$

To see this is a straightforward calculation as follows,

$$\operatorname{cov}(l_{1i}(\bar{\phi}_1), l_{2j}(\phi)) = \iint l_{1i}(\bar{\phi}_1) l_{2j}(\phi)\phi\ dy_1 dy_2 \quad = \int l_{1i}(\bar{\phi}_1)\left(\int l_{2j}(\phi)\phi\ dy_2\right) dy_1 = 0\ , \tag{B9}$$

where I have used again the fact that $L_2(\phi|a,\eta) = L_2(\phi_2|a,\eta)$.

To conclude, (B5) and (B6) imply (64) and (B5) and (B8) imply (65),

$$\begin{aligned}\Sigma L_{12}(\phi,\eta) &= \mathrm{cov}(L_1(\phi|a,\eta),L_2(\phi|a,\eta)) = \mathrm{cov}(L_1(\bar{\phi}_1|a_1,\eta)+L_1(\phi_2|a,\eta),L_2(\phi|a,\eta)) \\ &= \mathrm{cov}(L_1(\phi_2|a,\eta),L_2(\phi|a,\eta)) = \mathrm{cov}(L_1(\phi_2|a,\eta),L_2(\phi_2|a,\eta))\ .\ \square\end{aligned} \tag{B10}$$

Proof of Proposition 6. First, I show that the additional information has no value, if, and only if, $\Sigma R(\phi,y,z)=\Sigma R(\phi,y)$. By Corollary 2, $\Sigma R(\phi,y,z)=\Sigma R(\phi,y)$ implies the additional information has no value. By Proposition 2, the converse is also true if the marginal effort cost vectors span the entire space $\mathbb{R}^{m_1}\times\mathbb{R}^{m_2}$ for all actions that are implementable under both information systems. Thus, to complete the proof, it suffices to prove the following lemma.

Lemma 10 *The renegotiation sufficient statistic condition is equivalent to*

$$\Sigma L_1(\bar{\phi}_1,y_1,z_1)=\Sigma L_1(\bar{\phi}_1,y_1) \quad \textit{and} \quad \Sigma L_2(\phi_2,y,z)=\Sigma L_2(\phi_2,y)\ . \tag{B11}$$

In addition, if either one of the two equivalent conditions is satisfied, then

$$\Sigma L_{12}(\phi,y,z)=\Sigma L_{12}(\phi,y)\ . \tag{B12}$$

Proof of Lemma 10. Recall that for any two joint normally distributed signals, (y,z) is a sufficient statistic for y for all a if, and only if (see Proposition 6 in Christensen et al. 2010)

$$\Sigma L(\phi,y,z)=\Sigma L(\phi,y)\ . \tag{B13}$$

Assume that (B11) holds. Then, the sufficient statistic condition implies $\bar{\phi}_1(y_1,z_1|a_1)=\bar{h}_1(y_1,z_1)\bar{\phi}_1(y_1|a_1)$. By the same argument, $\phi_2(y,z|a)=h_2(y,z|a_1)\phi_2(y|a)$. Multiplying the two identities yields

$$\phi(y,z|a)=\bar{h}_1(y_1,z_1)h_2(y,z|a_1)\phi(y|a)\ . \tag{B14}$$

Thus, (84) is satisfied with $h(y,z|a_1) = \bar{h}_1(y_1,z_1)h_2(y,z|a_1)$. Because $h_2(y,z|a_1)$ is the density of the conditional distribution of z_2, given y_2, conditioned on a_1, y_1, z_1, it follows that $\int h_2(y,z|a_1)\ dz_2 = 1$. This proves (85), because $\int h(y,z|a_1)\ dz_2 = \bar{h}_1(y_1,z_1)$.

To prove the converse, assume the sufficient renegotiation statistic condition holds. A direct calculation shows that

$$\begin{aligned}\bar{\phi}_1(y_1,z_1|a_1) &= \int \phi(y,z|a)\ dy_2dz_2 \\ &= \int h(y,z|a_1)\phi(y|a)\ dy_2dz_2 = \int \left(\int h(y,z|a_1)\ dz_2\right)\phi(y|a)\ dy_2 \\ &= \bar{h}_1(y_1,z_1)\int \phi(y|a)\ dy_2 = \bar{h}_1(y_1,z_1)\bar{\phi}_1(y_1|a_1)\ .\end{aligned} \tag{B15}$$

A similar direct calculation shows that

$$\phi_2(y,z|a) = \frac{\phi(y,z|a)}{\bar{\phi}_1(y_1,z_1|a_1)} = \frac{h(y,z|a_1)}{\bar{h}_1(y_1,z_1)}\frac{\phi(y|a)}{\bar{\phi}_1(y_1|a_1)} = h_2(y,z|a_1)\phi_2(y|a)\ . \tag{B16}$$

But (B15) and (B16) are the sufficient statistic conditions for $\bar{\phi}_1(y_1,z_1|a_1)$, $\bar{\phi}_1(y_1|a_1)$ and $\phi_2(y,z|a)$, $\phi_2(y|a)$, respectively. Thus, (B15) and (B16) imply (B11).

It remains to prove (B12). Note that $\Sigma L_{12}(\phi,y,z) = \mathrm{cov}(L_1(\phi_2,y,z|a), L_2(\phi_2,y,z|a))$ and $\Sigma L_{12}(\phi,y) = \mathrm{cov}(L_1(\phi_2,y|a), L_2(\phi_2,y|a))$. The renegotiation sufficient statistic condition implies that $\phi_2(y,z|a) = h_2(y,z|a_1)\phi_2(y|a)$ with $\int h_2(y,z|a_1)\ dz_2 = 1$. A direct calculation shows that

$$\begin{aligned}L_1(\phi_2,y,z|a) &= \frac{\nabla_{a_1}\phi_2(y,z|a)}{\phi_2(y,z|a)} = \frac{\nabla_{a_1}h_2(y,z|a_1)\phi_2(y|a) + h_2(y,z|a_1)\nabla_{a_1}\phi_2(y|a)}{h_2(y,z|a_1)\phi_2(y|a)} \\ &= L_1(h_2(y,z|a_1)) + L_1(\phi_2,y|a)\ .\end{aligned} \tag{B17}$$

A similar direct calculation gives

$$L_2(\phi_2,y,z|a) = \frac{\nabla_{a_2}\phi_2(y,z|a)}{\phi_2(y,z|a)} = \frac{h_2(y,z|a_1)\nabla_{a_2}\phi_2(y|a)}{h_2(y,z|a_1)\phi_2(y|a)} = L_2(\phi_2,y|a)\ . \tag{B18}$$

Thus, to complete the proof it suffices to show that $\mathrm{cov}(L_1(h_2(y,z|a_1), L_2(\phi_2, y, z|a)) = 0$. Indeed, because $\int h_2(y,z|a_1)\ dz_2 = 1$,

$$\begin{aligned}&\int \frac{\nabla_{a_1} h_2(y,z|a_1)}{h_2(y,z|a_1)} \frac{\nabla_{a_2}\phi_2(y|a)}{\phi_2(y|a)} \phi(y,z|a)\ dydz \\ &= \int \left(\nabla_{a_1} \int h_2(y,z|a_1)\ dz_2\right) \nabla_{a_2}\phi_2(y|a) \bar{h}_1(y_1,z_1)\bar{\phi}_1(y_1|a_1)\ dz_1 dy_1 dy_2 = 0\ .\ \Box \end{aligned} \tag{B19}$$

This completes the proof of the Proposition. □

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