Disclosure Drifts in Investor Networks

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January 2011

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Abstract

This study develops a model of firm information diffusion in a setting where investors are linked in a social network. Because investors have finite attention and cognitive skills, the firm's disclosure initially reaches only a subset of the investor base. The remaining investors receive the information through the investor network. The structure of the investor network thus plays a critical role in determining investor trading patterns and the consequent price reaction to disclosure. In particular, investors who are most connected play a key role in driving the price response to disclosure, and the ability of the disclosure transmission mechanism to reach these investors becomes crucial. Our way of thinking offers a simple way to consolidate and extend the findings of several recent empirical papers that highlight the importance of the disclosure transmission mechanism (i.e., broadcast disclosures via press and electronic channels, or targeted disclosures via investor conferences and investor relations), and is a departure from the traditional accounting disclosure models that focus on the disclosed *information* itself and are largely silent on the properties of the underlying transmission *mechanism*.

1 Introduction

Traditional accounting disclosure models focus on the information content of disclosure, and have paid little attention to the transmission mechanism through which this disclosure is effected. However, empirical disclosure research has recently discovered rich and interesting empirical regularities on disclosure transmission media and venues. For example, Bushee and Miller (2010) find that hiring investor relation experts has a significant impact on investor interest in the firm and improves asset valuations; Bushee, Matsumoto, and Miller (2003) find that the type of conference call — open or closed — affects asset prices; likewise, Blankespoor, Miller, and White (2010) find that initiating or re-broadcasting primary disclosures in supplementary channels such as Twitter also move prices; finally, Bushee, Jung, and Miller (2010) find that the venue of disclosure — namely investor conferences — also has a significant impact on investor following and firm valuation. Similar interesting results carry over to disclosure in the business press, whose job more often is to *retransmit* information disseminated by the firm as opposed to original investigative reporting. Soltes (2010) finds that firms that are able to push their news into wider media outlets enjoy greater liquidity. Further, Li, Ramesh, and Shen (2010) examine a specific media outlet, namely Dow Jones Newswire, and find that firms whose disclosures the newswire retransmits enjoy significant price changes.

These findings on disclosure media and venues are at best curiosities in prior analytical studies, which typically focus on the information content of the disclosure rather than the disclosure mechanism.[1] There is a clear reason why this is so: price. Traders in these traditional models are resource unconstrained and have complete cognition. Once traders get information, they trade on it optimally (e.g., Kyle 1985). Moreover, if there is competition among the traders (e.g., Grossman 1976; Admati and Pfleiderer 1988b),

[1] For example, Grossman and Stiglitz (1980) and related models have informed traders incur a cost c to obtain information, but do not model the actual process by which the traders obtain the information or from whom the investors learn this information.

they trade on the information instantly. As a result, price immediately incorporates all the information in the disclosure, irrespective of how the disclosure is transmitted. Prices thus become the predominant channel through which investors communicate with one another.[2] However, once we grant that traders neither have unlimited wealth, nor complete cognition, the information content of price degrades, and other mechanisms of information transmission rise to prominence. This phenomenon is precisely what the empirical papers referred to in the previous paragraph document.

This paper models information transmission mechanisms in settings where price is no longer the key information conduit in the economy, and investors rely on other information sources such as social networks. The motivation for such models, well established in the economics and finance literature, is that investors have limited cognition and rely on trusted sources such as social and professional connections for credible information (see section Section 1.1 for a review). In the first step of our model, the firm makes an initial disclosure that reaches only a subset of investors. The limited audience for the disclosure could arise due to only some investors possessing the cognitive ability to process the value-relevant information in the disclosure (e.g., consider the case of a complex footnote disclosure) or due to other forms of investor limited attention problems (Hong, Stein, and Yu 2007). The initial information then slowly percolates through the investor network as investors spread the information to others with whom they are socially connected.[3] At periodic intervals investors receive limited wealth to invest (say from their excess wages). They then trade periodically with a rational market maker. The market maker's rational price setting mechanism then determines the dynamic trajectory of prices and the bid-ask spreads.[4]

[2] In fact, in standard asset pricing models, allowing investors to communicate with each other still leads to no or limited direct communication in equilibrium (e.g., Admati and Pfleiderer 1988a).

[3] For empirical evidence of this phenomena, see Hong, Kubik, and Stein (2004) and Hong, Kubik, and Stein (2005).

[4] Contrast this setup with the traditional accounting literature where the disclosure typically reaches some subset of investors (the "informed") who promptly process the value relevant information in the disclosure and trade in a wealth unconstrained manner until the price adjusts to the point where further

We characterize the flow of information and the resulting trading and pricing patterns. Our first set of results pertain first to how bid-ask spreads behave over time. Bid-ask spreads depend on the market maker's information disadvantage relative to traders which, in turn, depends on the strength of the market maker's beliefs and the fraction of traders who have superior information. In models that follow Glosten and Milgrom (1985), there is a fixed fraction of informed investors, and bid-ask spreads typically decline over time after an information event because the market maker learns from successive trades. The crucial difference in our setting is that because information flows directly through the investors' social ties, the number of informed investors changes with time. As a result, the effect of an increasing fraction of informed traders can dominate the effect of the market maker's learning from prices and cause an *increase* in the bid-ask spreads. This increase in bid-ask spread is temporary however; once a substantial fraction of the traders learn the information, the proportion of informed trades increases, thus reducing the market maker's post-trade information disadvantage and the consequent bid-ask spread.

The importance of information flow in investor network to the price mechanism suggests that firms could benefit from maximizing the flow of information through this network. Our second set of results therefore pertains to how firms design their disclosure mechanisms to maximize the flow of information. Note the crucial point here: our goal is not to model information *dissemination* — the firm can always put all its information on its website resulting in instaneous world wide dissemination. Our goal, instead, is to model information *insemination*, i.e., how investors who have limited attention and cognition *receive* the information.[5] Our model shows that the speed with which the population of investors becomes informed is increasing in the extent to which firms inseminate

trades are unprofitable. The remaining investors (the "uninformed") look to the price to rationally structure their limit or market orders. Prices are thus the key communicating channel. In our setting, prices lose this primacy because informed investors cannot trade large enough quantities to make prices a superior source of information. Investor networks then gain prominence as the communication channel.

[5] Also note in this regard that statutes such as Reg FD pertain to *dissemination* and not *insemination* of information, and therefore are largely irrelevant to our inquiry.

disclosures in highly connected investors. Even if a small fraction of highly connected investors disregards information about a firm, the network essentially ceases to function. Each investor is thus an island and the firm cannot rely on the network as a conduit for its disclosures. In such situations, we find that firms may attempt to inseminate a large group of investors simultaneously (e.g., Blankespoor, Miller, and White 2010), or use its investor relation apparatus to *build* investor networks (e.g., Bushee, Jung, and Miller 2010). This is a novel role of disclosure transmission practices that is missing in the current body of analytical disclosure research. Section 5 develops additional empirical implications of these findings.

1.1 Prior Literature

Our study is by no means the first to highlight patterns of information exchange in asset markets when price loses its role as the prime information conduit. A series of recent studies (e.g., Duffie, Gârleanu, and Pedersen 2005; Duffie and Manso 2007; Duffie, Malamud, and Manso 2009) model agents with limited resources who meet *randomly* in the marketplace for exchange. When they meet an appropriate counterparty, transactions and exchanges occur. These search and exchange dynamics then drive information exchange and pricing. These papers thus build on asset pricing papers such as Shiller and Pound (1989) that use basic epidemic models to model information diffusion when agents meet randomly.

The idea of random exchange is certainly true of several asset classes, but in other cases such as stocks, there is ample *empirical* evidence that investors rely on their social network to get information (e.g., Hong, Kubik, and Stein 2004, 2005).[6] Social networks

[6]The importance of social networks is evident in the Securities and Exchange Commission's recently opened investigations on both "affinity fraud" and "expert networks" to discipline individuals who misuse their social connections to defraud investors and take advantage of insider information (Holzer 2010; Zuckerman and Pulliam 2010). As another example, local shareholder concentration and communication facilitated Cedar Fair shareholders' coordinated resistance to the proposed sale to Apollo Global Management (Davidoff 2010).

have been also been implicated in the behavior of a variety of financial actors ranging from venture capitalists (Hochberg, Ljungqvist, and Lu 2010) and boards of directors (Hwang and Kim 2009) to banks (Leitner 2005). As a result, several prior studies have modeled asset pricing in investor networks (e.g. Colla and Mele 2010; Ozsoylev and Walden 2009).

A crucial difference between these asset pricing papers and ours is that these papers are primarily *static*. Information flows throughout the network in the first step, after which all investors trade simultaneously. As a result, the investors can be thought of collectively as one "big" investor who has information of a specific precision that depends on the network structure. Colla and Mele (2010) represent an investor network as a circular lattice to ease the computation of the precision of the aggregate trade. The network provides an initial endowment of information and they study price dynamics as investors continue to learn via prices, but not via the social network. Ozsoylev and Walden (2009) develop a model with a less structured network characterized by the fraction of individuals that have a given number of connections. They study a single round of trade in which pre-trade flow of information dictates the investors' information sets. These modeling choices are well-suited for these paper's variables of interest, namely prices, volume, volatility, liquidity, and trader welfare implications of network formation.

By contrast, our paper emphasizes disclosure *dynamics* in a setting that includes the direct communication of information in a social network. In our model, the firm's initial disclosure reaches a certain subset of investors and then diffuses throughout the network. Periodic trading commences simultaneously and prices and bid-ask spreads change with the information diffusion dynamics. Similar to Ozsoylev and Walden (2009), our network is also more unstructured than Colla and Mele (2010). Such heterogeneity in the investor network is not only more realistic, but is also essential to understand the findings of recent empirical accounting research on information dissemination mechanisms.

The remainder of the paper is organized as follows. Section 2 presents the basic model of an investor network, while Section 3 discusses how firm disclosures flow through the network. Section 4 models the market microstructure when this investor network trades with a market maker. Section 5 reports the empirical implications and Section 6 concludes.

2 A model of investor network

This section develops a representation of a network of investors. Our central assumption is that investors belong to social networks. We also assume that these investors have limited attention and cognitive abilities (Hong, Stein, and Yu 2007). Some investors may also find news from peers to be more credible than the firm's public disclosure. As a result, the firm's disclosure, while broadcast publicly, initially registers only with a fraction of investors. Subsequently, this information then spreads through the investor network (Hong, Kubik, and Stein 2004).[7]

The vastness of the literature on network configuration provides numerous ways to represent network structure (Newman 2010, Part IV). Fundamentally, a network contains *nodes* (i.e., individuals) and *edges* (i.e., connections between individuals). Given a set of N individuals, the network represents connections among those individuals. Graphically, each individual can be represented by a node and the *degree* of a node is the number of connections it has. A network can be characterized by its *degree distribution* where p_k denotes the fraction of individuals in the network with k connections to at most $N-1$ others and as few as zero. If we view the network as arising from the random establishment of connections between N individuals, the degree distribution can be thought of as the probability of individuals' creating a given number of connections. Table 1 lists the notation we use in this study.

(Insert Table 1 about here)

[7] We assume no variations in "inherent" investor types. Consistent with this assumption, empirical evidence finds that both individual (Shiller and Pound 1989; Ivković and Weisbenner 2007) *and* institutional (Shiller and Pound 1989; Hong, Kubik, and Stein 2005; Pareek 2009) investors are subject to influences from network effects.

Figure 1 provides an example of a network that includes 1,000 individuals. Figure 1, Panel A represents the degree distribution p_k. Panels B and C represent two networks generated from the degree distribution in Panel A. The degree distribution provides only a partial characterization of a network as is apparent from the fact that the networks in Panels B and C share the same degree distribution, yet are different from each other.

This use of degree distributions to characterize networks allows us to reflect the fact that individuals with more connections are more likely to receive information. This is an advantage over simple diffusion models that assume all individuals are the same. The shortcoming is that the use of degree distributions imposes homogeneity within degree. For example, any individual with 20 connections is just as likely to receive information as any other individual with 20 connections. This rules out situations where one individual with 20 connections may be closer to information 'hubs' than others.

Two summary measures of the network are the average number of connections (average degree) $\mathrm{E}[k] = \sum_k p_k k$ and heterogeneity $\kappa = \mathrm{E}[k^2]/\,\mathrm{E}[k]$. The networks in the examples in Figure 1 have average degree $\mathrm{E}[k] = 1.7$ and heterogeneity $\kappa = 5.8$. In other words, the average individual in the networks has 1.7 connections to others and the average squared connection is 5.8 times larger than the average degree, or $\mathrm{E}[k^2] = \kappa \times \mathrm{E}[k] = 5.8 \times 1.7 = 9.9$.

(Insert Figure 1 about here)

Both networks in Figure 1 exhibit a large cluster containing roughly half of the individuals in the population. An important notion in understanding how information will flow through the network is that of a *giant component* that, by definition, 'scales up' with the size of the network. The large clusters in Figure 1 are examples of the portion of the network included in a giant component that scales up as the population $N \to \infty$. The isolated clusters of individuals in Figure 1 are examples of *small components*, each of which becomes a vanishingly small portion of the population as $N \to \infty$.

The following proposition, first proved by Molloy and Reed (1995), indicates the conditions for the existence of a giant component where $x_1 = \mathrm{E}[k]$ denotes the average number of first neighbors in the network, x_2 denotes the average number of second neighbors (neighbors of neighbors), and so on.

Proposition 1. *Denote the average number of* d^{th} *neighbors,* $d \in \{1, 2, ...\}$*, by* x_d*. Then* $x_d = (x_2/x_1)^{d-1}x_1 = (\kappa - 1)^{d-1}\,\mathrm{E}[k]$ *and the network has a giant component if and only if* $\kappa > 2$*.*

The heterogeneity κ determines whether a giant component will exist, with the condition being that $\kappa > 2$. The condition $\kappa > 2$ is equivalent to the average number of second neighbors exceeding the average number of first neighbors ($x_2 > x_1$). Intuitively, the existence of a giant component depends only on the ratio x_2/x_1 because if the average number of people that can be reached in two links grows for a given individual, then consider that individual's third neighbors. These are just the second neighbors of the individuals immediate neighbors. Because $x_2 > x_1$, there are more of these, on average, than the number of first neighbors. If $\kappa > 2$, then the number of d^{th} neighbors increases exponentially with distance d indicating that an infinite number of linkages allows an individual to reach the infinite number of individuals in the giant component (Newman 2010, Section 13.5). If $\kappa < 2$, then x_d declines exponentially with d indicating that every individual in every component of the network can reach all other individuals in their components within a finite number of linkages. This is only possible if the components have a finite size.

In the context of disclosure, both the presence and size of the giant component are important determinants of the extent of information flow. Unconnected investors neither obtain information shared amongst those in the giant component nor pass along information outside of their small circle of relationships. If there is no giant component, each cluster of investors is essentially an island with no communication to others except via price. These isolated investors receive information only through direct disclosures from the firm or direct effort by the individual such as obtaining information from media outlets or individual research. Information thus does not quickly disperse among investors in small components so that they contribute little to the informativeness of prices. If there is a giant component in the network and individuals in that group have information about the firm, that information will spread both directly through word-of-mouth and indirectly as an increasingly informed investor base makes trades that yield increasingly informative prices. We now turn our focus to this flow of information.

3 Flow of disclosure through the network

The power of our model is driven by the notion that information flows through a network of individuals. This implies that individuals pass along information and that, crucially, this does not occur instantaneously. In this section, we model this flow of information through the network, ultimately deriving a specification of the fraction of informed investors an any time, t. The basic structure follows the Barthélemy, Barrat, Pastor-Satorras, and Vespignani (2004) epidemic model in which individuals are distinguished solely based on their degree. We assume that the network is *uncorrelated*, which means that there is no correlation between an individual's degree and the degrees of its neighbors. This rules out, for example, highly connected people more frequently associating with other highly connected people. The value of imposing these restrictions is that they allow us to derive analytical results that are unavailable when working with general network structures.[8]

Using this basic model, we next establish the model's parameters. In modeling the flow of information through the network, we use the notation I_{kt} to denote the number

[8]Many real-world networks exhibit assortative correlation (highly connected nodes connected with other highly connected nodes) (Watts and Strogatz 1998) and we make the assumption of an uncorrelated network purely for analytical convenience. Networks with degree correlation continue to have giant components and Brede and Sinha (2005) find that assortative correlation makes epidemics (or, disclosure in our setting) spread more quickly.

of individuals with degree k that have received the information by time t. The portion of degree k individuals who are informed is $\mu_{kt} = I_{kt}/N_k$ where N_k denotes the number of individuals with degree k. The parameter λ characterizes how fast information flows through the network and governs the probability $\lambda\,\mathrm{d}t$ that information is transmitted from an informed individual in a short time period. Lastly, the expected portion of a degree k individual's neighbors that are informed at time t is given by the Θ_{kt}. Given these parameters, the portion μ_{kt} of degree k individuals that are informed evolves as follows:

$$\frac{\mathrm{d}\mu_{kt}}{\mathrm{d}t} = \lambda \underbrace{(1-\mu_{kt})}_{\substack{\text{Portion of}\\ \text{degree } k\\ \text{uninformed}}} \underbrace{k\Theta_{kt}}_{\substack{\text{Expected}\\ \text{number of}\\ \text{informed}\\ \text{neighbors}}} . \tag{1}$$

In the special case of an uncorrelated network in which the degree of an individual is uncorrelated with the degree of his neighbors, $\Theta_{kt} = \Theta_t$ for all k.[9] The resulting density of informed neighbors is:

$$\Theta_t = \frac{1}{\mathrm{E}[k]} \sum_{k=1}^{N-1} (k-1) p_k \mu_{kt}. \tag{2}$$

In solving for the dynamics of the early stages of information flow, the portion of informed individuals is governed by the following equations from (1) and (2) in which we ignore terms of order μ^2:

$$\frac{\mathrm{d}\mu_{kt}}{\mathrm{d}t} = \lambda(1-\mu_{kt})k\Theta_{kt} = \lambda k \left(\Theta_t - \frac{1}{\mathrm{E}[k]} \sum_{s=1}^{N-1} (s-1) p_s \mu_{kt} \mu_{st} \right) \approx \lambda k \Theta_t, \tag{3a}$$

$$\frac{\mathrm{d}\Theta_t}{\mathrm{d}t} = \frac{1}{\mathrm{E}[k]} \sum_{k=1}^{N-1} (k-1) p_k \frac{\mathrm{d}\mu_{kt}}{\mathrm{d}t} \approx \frac{1}{\mathrm{E}[k]} \sum_{k=1}^{N-1} (k-1) p_k \lambda k \Theta_t = \lambda(\kappa - 1)\Theta_t, \tag{3b}$$

where $\kappa = \mathrm{E}[k^2]/\,\mathrm{E}[k]$. The crucial approximation is the neglecting of terms of order

[9]Models of correlated networks must be solved numerically. For example, Boguñá, Pastor-Satorras, and Vespignani (2003) provide some description of the spread of epidemics in a model that allows only the specification of 'first-order' correlations (probability that a degree k individual is linked to a degree k' individual, but ignoring correlation in the second-neighbor links).

μ^2 in (3a). The model becomes intractable when including higher order terms and the assumption can be justified by a focus on the early stages of the spread of a disclosure so that the μ_t terms are small. We later study the targeted release of information and, in particular, show that firms will wish to target highly connected individuals. As a result, the fraction initially informed μ_{k0} need not be small in a targeted disclosure setting. The approximation still likely holds in our context, however. So long as the likelihood p_k of a highly connected individual is small, which holds, for example, in many empirically observed networks with few 'hubs' and many 'spokes' (Jackson 2008, Chapters 3 and 5), this approximation still holds because the assumption of an uncorrelated network implies that a highly connected individual is unlikely to be connected to another highly connected individual due to their scarcity in the broad population.

We next solve the differential equation $\frac{1}{\Theta_t}\,\mathrm{d}\Theta_t = \lambda(\kappa-1)\,\mathrm{d}t$ from (3b). This yields the solution:

$$\Theta_t = \mathrm{e}^{\lambda(\kappa-1)t}\,\Theta_0. \tag{4}$$

Substituting (4) into (3a) yields the differential equation $\mathrm{d}\mu_{kt} = \lambda k\,\mathrm{e}^{\lambda(\kappa-1)t}\,\Theta_0\,\mathrm{d}t$ and the solution:

$$\mu_{kt} = \mu_{k0} + \frac{k}{\kappa-1}(\mathrm{e}^{\lambda(\kappa-1)t} - 1)\Theta_0. \tag{5}$$

Using $\Theta_0 = \frac{1}{\mathrm{E}[k]}\sum_k (k-1)p_k\mu_{k0}$ (from equation (2)), we have the average fraction informed μ_t as follows:

$$\begin{aligned}\mu_t &= \sum_{k=1}^{N-1}\mu_{kt}p_k = \sum_{k=1}^{N-1}\left(\mu_{k0} + k\frac{\mathrm{e}^{\lambda(\kappa-1)t}-1}{\kappa-1}\Theta_0\right)p_k \\ &= \mu_0 + \frac{\mathrm{e}^{\lambda(\kappa-1)t}-1}{\kappa-1}\sum_{k=1}^{N-1}(k-1)p_k\mu_{k0} \\ &= \sum_{k=1}^{N-1}\left(1 + (k-1)\frac{\mathrm{e}^{\lambda(\kappa-1)t}-1}{\kappa-1}\right)p_k\mu_{k0}.\end{aligned} \tag{6}$$

We utilize the fraction μ_t of informed investors as the metric of the extent to which information has flowed through the population of investors. In the next section we discuss how disclosure and network structure impacts the flow of information which, in turn, impacts bid-ask spreads.

4 Market microstructure pricing

In this section we embed the flow of information in a Glosten and Milgrom (1985)-type market microstructure asset pricing model. To do so, we first need to characterize the nature of information transmitted. As in Diamond and Verrecchia (1987) and Easley, Kiefer, OHara, and Paperman (1996), we consider shares in a single risky firm with payoff $\tilde{v}$ and a disclosure or news event that could be either good or bad news. We assume that all parties are risk-neutral and denote the values $\bar{v} \equiv \mathrm{E}[\tilde{v}|\text{Good news}]$ and $\underline{v} \equiv \mathrm{E}[\tilde{v}|\text{Bad news}]$. The information transmitted through the investors network is repetition of the initial disclosure, i.e., “Good News” or “Bad News”.

At discrete time interval t, a randomly selected trader is allowed to trade one share. The selection of the particular investor is uniform over the population (i.e., the trading shock is independent of the information flow in the investor network). As a result, the trading investor is informed with probability μ_t based on the flow of information through the network. An informed investor buys a share if the firm value is high and sells if it is low. An uninformed investor buys or sells a share with equal probability which could be due to a liquidity shock as in Diamond and Verrecchia (1987).[10]

This investor places his trade with the market maker. The market making process follows a variant of Glosten and Milgrom (1985). Trades occur sequentially and the market maker sets prices at breakeven levels. As in Glosten and Milgrom (1985), the

[10]The uninformed investors serve the role of ‘noise traders’ in that they purchase (sell) shares at the ask (bid) price, which is greater (less) than their own expected value since the ask price incorporates the possibility that the trade arises from an informed investor. Note that we assume that informed investors do not have such liquidity shocks. In principle, trade is driven by a combination of information and liquidity shocks, but incorporating both of these forces simultaneously for each trader introduces additional complexity that would only detract from developing the intuition of how the flow of information through the network affects spreads.

market maker's information set consists of a prior belief and the series of trades, i.e., the market maker only looks at order flows and is not privy to the information that investors have.

We denote by $\mathrm{E}_{t-1}[\tilde{v}]$ the market maker's expected value of $\tilde{v}$ given trades observed prior to the discrete time t. Given the breakeven condition, the bid and ask prices are:

$$\text{Ask: } a_t = \mathrm{E}_{t-1}[\tilde{v}|\text{Buy order}_t] = \underline{v} + (\bar{v} - \underline{v})\,\mathrm{P}_{t-1}(\bar{v}|\text{Buy order}_t) \tag{7a}$$

$$\text{Bid: } b_t = \mathrm{E}_{t-1}[\tilde{v}|\text{Sell order}_t] = \underline{v} + (\bar{v} - \underline{v})\,\mathrm{P}_{t-1}(\bar{v}|\text{Sell order}_t). \tag{7b}$$

The conditional probabilities in (7) depend on the market maker's prior beliefs and the population of informed traders. The proportion of informed traders versus the overall population of investors is given by μ_t in (6). In other words, the actual dynamics of information flow match the expected dynamics.[11] Figure 2 illustrates the likelihoods of buys and sells at time t.

(Insert Figure 2 about here)

The following proposition establishes the bid price b_t and ask price a_t for time t trades where $p_{t-1} \equiv \mathrm{P}_{t-1}(\bar{v})$ denotes the market maker's belief prior to observing the time t trade:

Proposition 2. *Assume that the actual fraction of informed investors equals the expected fraction μ_t. The bid and ask prices for time t trades are:*

$$b_t = \underline{v} + (\bar{v} - \underline{v})\underbrace{\frac{p_{t-1}(1-\mu_t)/2}{(1-p_{t-1})\mu_t + (1-\mu_t)/2}}_{\leq p_{t-1}} \qquad a_t = \underline{v} + (\bar{v} - \underline{v})\underbrace{\frac{p_{t-1}\mu_t + p_{t-1}(1-\mu_t)/2}{p_{t-1}\mu_t + (1-\mu_t)/2}}_{\geq p_{t-1}}. \tag{8}$$

[11]See DeMarzo, Vayanos, and Zwiebel (2003) for a discussion of the complexity involved with updating beliefs in a setting where information flows through a network and arguments in favor of assuming that individuals obey heuristic rules.

The time t bid-ask spread is:

$$s_t = a_t - b_t = (\bar{v} - \underline{v}) \frac{4p_{t-1}(1-p_{t-1})\mu_t}{1-(1-2p_{t-1})^2\mu_t^2}. \tag{9}$$

A buy (sell) order can originate from an uninformed trader or an informed trader with good (bad) news; hence, the market maker's belief that there was good news increases (decreases) with buy (sell) orders. The following proposition states how prices are affected by the market maker's uncertainty regarding the flow of information:

Proposition 3. *The bid-ask spread s_t is symmetric and concave in p_{t-1} with a peak at $p_{t-1} = 1/2$. The spread is increasing and convex in the fraction of informed investors μ_t. If the flow of information is random, then the bid and ask prices are closer to $\underline{v} + (\bar{v} - \underline{v})/2$ than in* (8)*. The bid ask spread s_t is higher than in* (9)*.*

The spread reflects the market maker's informational disadvantage relative to traders. The spread is highest at $p_{t-1} = 1/2$ where the market maker is most unsure about the firm's value.[12] The spread is symmetric in p_{t-1} around this point and the concavity in p_{t-1} reflects the decline in spreads as the market maker learns more about firm value. The spread is also increasing in μ_t (Glosten and Milgrom 1985, Proposition 5ii) and convex in μ_t, which is because a higher portion of informed investors makes each trade more informative. Figure 3, Panel A plots the bid-ask spread s_t for time t trades as a function of the market maker's prior p_{t-1}, illustrating the concavity in p_{t-1}. Figure 3, Panel B plots the bid-ask spread as a function of the fraction μ_t of informed investors, illustrating the convexity in μ_t. Uncertainty about the fraction of informed traders widens spreads because it increases the market maker's informational disadvantage relative to traders who know whether or not they are informed.

(Insert Figure 3 about here)

[12]This can also be derived from Diamond and Verrecchia (1987, Corollary 3) by assuming zero costs of short sales.

In most models, the portion of informed traders remains constant so that the spread declines over time, on average, as the market maker learns more from trades and p_{t-1} approaches either zero or one. The following proposition states that this can differ in settings where information flows over a network so that the portion of informed investors grows over time.

Proposition 4. *If the fraction of informed investors is constant, then bid-ask spreads decline over time:* $\mathrm{E}[s_t] < \mathrm{E}[s_{t-1}] < \cdots < \mathrm{E}[s_1]$. *If the fraction of informed investors increases by a sufficiently large amount between trades, the bid-ask spread can increase.*

When there is a constant fraction of informed investors, spreads decline as the market maker learns more from trades, thus losing his information disadvantage. However, when information flows over time, the fraction of informed traders increases over time, which acts to increase spreads. In the early period following a disclosure, the fraction of informed investors grows exponentially as shown in expression (6). Thus, the effect of increasing informed investors can outpace the effect of the market maker's learning and increase bid-ask spreads. Later, as the network of investors becomes saturated with information, the growth slows and spreads will again decline.

Relating back to Section 2, the long run behavior of bid-ask spreads depends crucially on the size of the giant component in the network. If the giant component occupies a large portion of investors, then trades become highly informative when the information has reached all of the giant component. If the giant component represents a small portion of investors, however, trades remain noisy signals of value even after information has reached all connected investors. In this latter case, spreads may remain high for an extended period of time (unless the firm initially inseminates a large proportion of investors).

5 Empirical implications

Thus far, we have established how firm disclosures may flow through a network of investors and the resulting analytical implications of this information flow for a market maker's price-setting. Fundamentally, in a setting in which investors receive information via their network of contacts, the shape of the network impacts the rate of information flow which, in turn, affects the proportion of informed investors, ultimately resulting in pricing effects. These analytical results yield two immediate empirical implications regarding: 1) how a firm begins the information flow (i.e., "seeds" the network with targeted disclosure), and; 2) how the shape of the network impacts the information cascade.

5.1 Targeted disclosure

An important insight from the empirical studies of Bushee and Miller (2010), Bushee, Matsumoto, and Miller (2003) and Blankespoor, Miller, and White (2010) is that disclosure dissemination is the not the same as as disclosure insemination (or investor receipt of disclosure). In traditional disclosure models, this distinction is moot because investors have complete cognition, no resource constraints, and can thus price the disclosure quickly. However, our model makes this distinction explicitly, and thus speaks to the role of investor relations more explicitly.[13]

An important role of corporate investor relations departments is to ensure that the investor base receives the information. In a disclosure setting where each individual contact may be costly, companies may gain efficiencies by targeting key individuals who can trigger a cascade of information through a large population of investors. In order to understand how an initial contact with an individual affects the portion μ_t of the

[13]As noted before, regulatory statues such as Reg FD pertain to disclosure dissemination and thus only of marginal relevance to this study. In any event, the systematic effectiveness of Reg FD obviously depends on regulatory enforcement, the systematic pervasiveness of which is an open empirical question (e.g., Holzer 2010).

population that is informed, substitute $p_k = N_k/N$ and $\mu_{k0} = I_{k0}/N_k$ into (6) to yield:

$$\mu_t = \sum_{k=1}^{N-1} \left(1 + (k-1)\frac{\mathrm{e}^{\lambda(\kappa-1)t} - 1}{\kappa - 1}\right) \frac{I_{k0}}{N}. \tag{10}$$

In terms of increasing the fraction of the population that is informed, the benefit of disclosing to an additional individual with k connections can be represented by incrementing I_{k0} by one. From (10), we observe that this increases the fraction informed μ_t by:

$$\left(1 + (k-1)\frac{\mathrm{e}^{\lambda(\kappa-1)t} - 1}{\kappa - 1}\right) \frac{1}{N}. \tag{11}$$

Because $\kappa > 1$, (11) is increasing in the degree k, which yields the following prediction:

Prediction 1. *The value of initially disclosing to an individual, as measured by the impact on the average portion informed μ_t, is increasing in the individual's degree k.*

The implication of Prediction 1 is that firms can more quickly reach the population of investors by specifically identifying and targeting highly connected individuals in their disclosure activities. These individuals are highly connected and thus can quickly spread information to others. For example, public companies in the UK hold monthly meetings in the City of London with large institutional investors (Bauguess, Dunigan, Park, McGurn, Chew, and Walkling 2010). Our paper shows why such activities might be useful supplements to regular disclosure practices.

More interestingly, the idea that all investors do not receive disclosure simply because the firm has broadcast it also has implications for an important empirical phenomena in accounting research, namely the post earnings announcement drift (PEAD). This is an extensively studied phenomenon whose cause is still far from being settled. The recent empirical literature on the subject has focused on which *types* of investors cause PEAD (Bartov, Radhakrishnan, and Krinsky 2000; Hirshleifer, Myers, Myers, and Teoh 2008; Taylor 2010). If highly connected investors largely initially disregard a firm's earnings release, information may slowly reach the market. Our model suggests that future empirical studies should jointly investigate not only the investor type, but also investors' connections within networks that disseminate information about the firm.

5.2 Network structure

We have shown that the structure of connections between investors impacts the flow of information, particularly through heterogeneity in the number of connections. Network heterogeneity κ impacts information dynamics via the term $\frac{e^{\lambda(\kappa-1)t}-1}{\kappa-1}$ in (10) and (11). This expression is increasing in κ, which yields the following prediction:[14]

Prediction 2. *The value of initially disclosing to an individual, as measured by the impact on the average portion informed μ_t, is increasing in the network's heterogeneity κ because a high κ implies the existence of highly connected individuals who accelerate the flow of information. Furthermore, the value of disclosing to highly connected individuals (high degree k) is increasing in network heterogeneity.*

The intuition behind Prediction 2 is that network heterogeneity is associated with the presence of 'hubs' in the network. Prediction 2 provides support for the notion of firms sponsoring investor relations conferences that also serve as networking events. Networking amongst highly connected investors can serve to create 'super-hubs' that facilitate the spread of information (Bushee, Jung, and Miller 2010).

In analytical terms, network heterogeneity is driven by the distribution of p_k. Empirically, an important class of networks are those that follow power laws, so called because $p_k \propto k^{-\gamma}$. Many real world social networks approximately follow power law distributions with exponents between 2 and 3 (Newman 2010, Chapter 8). The heterogeneity parameter κ becomes infinite for power law distributions with $\gamma < 3$, which implies that information can spread extremely quickly (Barthélemy, Barrat, Pastor-Satorras, and Vespignani

[14]See Appendix for proof.

2004). The parameter γ also determines the size of the giant component, which is the fraction of investors that ultimately become informed if disclosure is initiated with investors that are part of the giant component. Figure 4 illustrates how the size of the giant component varies with γ. For $\gamma \leq 2$, the giant component occupies the entire network. As γ becomes larger, there are fewer highly connected hubs in the network and the size of the giant component declines. No giant component exists for $\gamma > 3.48$ (Aiello, Chung, and Lu 2000). Given the empirically common range of $2 < \gamma < 3$, it is highly likely that the social network of investors contains a giant component. This suggests that companies can inseminate a large fraction of the investor population by initiating a cascade of information that relies on social networks for its transmission.

(Insert Figure 4 about here)

Power law networks also exhibit what has been dubbed a robust-yet-fragile property. Specifically, the random elimination of nodes in power law networks is unlikely to affect their functioning, making them robust. A company disclosure may still reach a wide audience if a random set of investors fails to pass along information. Figure 5, Panel A depicts this type of scenario. The deletion of nodes from the network corresponds to individuals failing to pass along information since such individuals are effectively absent from the network for purposes of transmitting disclosures. We plot the effect of deleting nodes on the size of the giant component for different values of the exponent γ for networks with a power law degree distribution.[15] The networks maintain a giant component even after nearly all of the vertices are deleted.

In contrast, the 'fragile' side of the robust-yet-fragile property occurs because the removal of *highly connected* nodes in a power law network dramatically reduces the size of the giant component. Figure 5, Panel B depicts the effect of removing nodes, beginning

[15] We determine the size of the giant component using the techniques in Newman (2010, Section 16.2) for random deletion and Newman (2010, Section 16.3) for targeted deletion.

with the most highly connected nodes. The removal of less than 3% of nodes eliminates the giant component in all three examples in the empirically common range of $2 < \gamma < 3$. In other words, if the 3% of individuals with the most social connections ignore information about a company, it can be as if the network of investors has no giant component and that company cannot rely on social networks as a means of reaching a nontrivial fraction of the population.

(Insert Figure 5 about here)

A necessary condition for a delayed reaction to price is that trades fail to quickly reveal firm value. Also, a large enough proportion of investors must become informed in order for market makers to learn from trades. In a power law network, Prediction 2 implies that there will be a rapid spread of information for sufficiently low γ; however, if highly connected nodes fail to participate in the passing of information, this will both reduce the κ parameter that governs the speed of information flow and the size of the giant component that ultimately becomes informed. If either γ is high, indicating that investors have few connections, or highly connected investors fail to pass along information, then relatively few investors will become informed. The following prediction summarizes these facts.

Prediction 3. *If the network of investors follows a power law, price drifts only occur in settings where either investors have few connections (high γ) or if highly connected individuals abstain from participation in the information network.*

As an example of Prediction 3, if highly connected individuals only invest time and resources in learning and passing along information about large or highly liquid firms, this could be a reason why we do not observe price drifts for such firms. For example, Chorida, Subrahmanyam, and Tong (2010) find that liquid stocks exhibit none of the anomalies for earnings drift, momentum or accruals. This is consistent with highly connected investors focusing on liquid firms and thus ensuring rapid incorporation of information into prices.

6 Conclusion

The impact of information dissemination mechanisms on pricing is a research area in which empirical findings are ahead of theoretical foundations. A crucial point these papers make is that information dissemination is not the same as information insemination. That is, just because the firm has broadcast a disclosure does not mean that the investor base has processed and priced it. As a result, disclosure practices' effectiveness in reaching investors is crucial for the firm's stock prices (e.g., Soltes 2010; Bushee, Jung, and Miller 2010). In this paper, we ground this idea into an analytical framework. Specifically, we provide a model that derives asset prices when investors receive information from their social network, and then trade with a rational market maker. The ensuing price dynamics depend crucially on both the network structure as well as the firm's ability to control the pattern of the initial insemination of information into the investor base. Our model squares well with recent investor communication innovations such as Twitter (Blankespoor, Miller, and White (2010)) and Extensible Business Reporting Language (XBRL), which are fundamentally changing the way in which firms can communicate with investors. More strikingly, these technological advances have the power to help firms *alter* the network structures of their investor base, a feature our model shows can have powerful disclosure ramifications. These possibilities create intriguing empirical opportunities based, in part, on the analytical foundations established in this paper.

We close this study with a historical perspective. At its most abstract, the notion of becoming informed can be modeled as changing one's state (i.e., from an uninformed state to an informed state). When there is no single overriding information conduit such as price, the ambient background assumes prominence as the information transmission mechanism. Understanding this problem occupies a signature place in modern physics and mathematics, which finance and economic studies are now adapting to their own uses. The idea of agents randomly meeting each other in the marketplace is isomorphic to the study of heat transmission of gases, whose molecules collide with each other randomly. Likewise, the mathematics of structured and random networks has its origins in heat transmission in structured lattices such as crystals and amorphous substances such as gels (e.g., Frish 1996; Stauffer and Aharony 1994). These mathematical tools have since been adopted in a wide variety of networked settings ranging from biology, ecology, and epidemiology to telecommunications networks, Internet search, and sociology (e.g., Newman 2010, Part I). In the context of research on financial markets, the increasingly frequent departures from the notion of efficient prices suggest that these mathematical tools will only increase in importance.

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Appendix

Proof of Proposition 1

The proof follows the approach in Newman (2010, Section 13.5) (Also see Molloy and Reed 1995). We first define the generating functions for the distribution p_k and the excess degree (degree of a neighbor in excess of the link to a given individual) $q_k = (k+1)p_{k+1}/\operatorname{E}[k]$:

$$g_0(z) \equiv \sum_{k=0}^{\infty} p_k z^k \qquad g_1(z) \equiv \sum_{k=0}^{\infty} q_k z^k = \frac{1}{\operatorname{E}[k]} \sum_{k=0}^{\infty} (k+1) p_{k+1} z^k = \frac{g_0'(z)}{g_0'(1)}. \tag{A.1}$$

The application of an induction argument shows that the generating functions for the probability that the d^{th} neighbor has degree k are:

$$f_2(z) = g_0\left(g_1(z)\right) \qquad f_3(z) = g_0\left(g_1\left(g_1(z)\right)\right) = f_2\left(g_1(z)\right) \quad \cdots \quad f_d(z) = f_{d-1}\left(g_1(z)\right). \tag{A.2}$$

By definition, given a generating function $g(z)$, the mean under the corresponding probability is $g'(1)$. Thus:

$$x_d = \left.\frac{\mathrm{d}f_d(z)}{\mathrm{d}z}\right|_{z=1} = \left.\frac{\mathrm{d}f_{d-1}(z)}{\mathrm{d}z}\right|_{z=1} g_1'(1) = x_{d-1} g_1'(1). \tag{A.3}$$

Iterating then gives:

$$x_d = x_{d-2} g_1'(1)^2 = \cdots = x_{d-k} g_1'(1)^k = \cdots g_1'(1)^{d-1} x_1. \tag{A.4}$$

The average number of neighbors $x_1 = \operatorname{E}[k]$ while $g_1'(1) = \kappa - 1$. If there is a giant component, which by definition contains an infinite number of individuals, then the number of d^{th} neighbors must become infinite as the distance d increases. ■

Proof of Proposition 2

The derivations of the bid and ask prices follow directly from applying Bayes' rule. The inequalities follow because $0 \leq p_{t-1} \leq 1$ and are strict only when the market maker is entirely sure of $\tilde{v}$'s value (p_{t-1} equals zero or one). ■

Proof of Proposition 3

The first part follows from direct computations:

$$\begin{aligned}
\frac{\partial s_t}{\partial p_{t-1}} &= (\bar{v} - \underline{v})\frac{4\mu_t(1-\mu_t^2)}{[1-(1-2p_{t-1})^2\mu_t^2]^2}(1-2p_{t-1}) \propto 1-2p_{t-1} \\
\frac{\partial^2 s_t}{\partial p_{t-1}^2} &= -(\bar{v} - \underline{v})\frac{8\mu_t(1-\mu_t^2)[1+3(1-2p_{t-1})^2\mu_t^2]}{[1-(1-2p_{t-1})^2\mu_t^2]^3} < 0 \\
\frac{\partial s_t}{\partial \mu_t} &= (\bar{v} - \underline{v})\frac{4p_{t-1}(1-p_{t-1})[1+(1-2p_{t-1})^2\mu_t^2]}{[1-(1-2p_{t-1})^2\mu_t^2]^2} > 0 \\
\frac{\partial^2 s_t}{\partial \mu_t^2} &= (\bar{v} - \underline{v})\frac{8\mu_t p_{t-1}(1-p_{t-1})(1-2p_{t-1})^2[3+(1-2p_{t-1})^2\mu^2]}{[1-(1-2p_{t-1})^2\mu_t^2]^3} > 0.
\end{aligned} \tag{A.5}$$

Direct computations show that the $\mathrm{P}_{t-1}(\bar{v}|\mathrm{Buy}_t)$ and $\mathrm{P}_{t-1}(\bar{v}|\mathrm{Sell}_t)$ terms in (7) are concave (convex) in μ_t for p_{t-1} greater than (less than) 1/2 since the second partial derivatives of both are proportional to $1 - 2p_{t-1}$:

$$\begin{aligned}
\frac{\partial\, \mathrm{P}_{t-1}(\bar{v}|\mathrm{Buy}_t)}{\partial \mu_t} &= (1-2p_{t-1})\frac{4p_{t-1}(1-p_{t-1})}{[1-(1-2p_{t-1})\mu_t]^3} \propto 1-2p_{t-1} \\
\frac{\partial\, \mathrm{P}_{t-1}(\bar{v}|\mathrm{Sell}_t)}{\partial \mu_t} &= (1-2p_{t-1})\frac{4p_{t-1}(1-p_{t-1})}{[1+(1-2p_{t-1})\mu_t]^3} \propto 1-2p_{t-1},
\end{aligned} \tag{A.6}$$

where the proportionality follows form $0 \leq p_{t-1} \leq 1$ and $0 \leq \mu_t \leq 1$. Thus, for a given prior p_{t-1}, Jensen's inequality implies that the post-trade probabilities $\mathrm{P}_{t-1}(\bar{v}|\mathrm{Buy}_t)$ and $\mathrm{P}_{t-1}(\bar{v}|\mathrm{Sell}_t)$ will be lower (higher) than in (7) when the prior p_{t-1} is greater than (less than) 1/2. This pushes the posterior beliefs closer to 1/2. This applies at all times t so that the prior beliefs at any time $t-1$ are closer to 1/2 than implied when the market maker knows the flow of information.

When analyzing the effect of uncertainty about the information flow on bid-ask spreads, first, ignoring any impact of uncertainty on p_{t-1}, note that s_t is convex in μ_t so that Jensen's inequality implies that uncertainty about the actual flow of information will increase spreads relative to those given in (9) based on the expected flow of information. When considering the effect of via p_{t-1}, note that s_t is increasing in p_{t-1} if and only if p_{t-1} is less than 1/2. From the first part, we know that p_{t-1} will be closer to 1/2 when there is uncertainty about μ_t, which will further increase the spread beyond the direct effect from the convexity in s_t in μ_t. ■

Proof of Proposition 4

Set $\mu_t = \mu$ for all t and consider an arbitrary time period t. The spread for $t+1$ trades is given by (9) and is known at time t:

$$s_{t+1} = \mathrm{E}_t[s_{t+1}] = \frac{4p_t(1-p_t)\mu}{1-(1-2p_t)^2\mu^2}. \tag{A.7}$$

Now consider the time $t-1$ expectation of the $t+1$ spread:

$$\mathrm{E}_{t-1}[s_{t+1}] = \mathrm{E}_{t-1}\left[\frac{4p_t(1-p_t)\mu}{1-(1-2p_t)^2\mu^2}\right] < \frac{4p_{t-1}(1-p_{t-1})\mu}{1-(1-2p_{t-1})^2\mu^2} = s_t = \mathrm{E}_{t-1}[s_t], \tag{A.8}$$

where the inequality follows from Jensen's inequality (See (A.5) for the concavity of s_{t+1} in p_t) and rational updating $\mathrm{E}_{t-1}[p_t] = p_{t-1}$. The unconditional expectation of the difference in spreads from t to $t+1$ is then:

$$\mathrm{E}[s_{t+1}-s_t] = \mathrm{E}\left[\mathrm{E}_{t-1}[s_{t+1}-s_t]\right] = \mathrm{E}\left[\mathrm{E}_{t-1}[s_{t+1}]-s_t\right] < 0, \tag{A.9}$$

where the inequality follows from (A.8) since the expected change in spread is negative for all $t-1$ prior beliefs. While updating tends to reduce spreads, (A.5) shows that spreads are increasing in the fraction of informed investors. As a result, a sufficiently large increase in the fraction of informed investors can dominate the effect of updating and cause an increase in spreads. ■

Proof of Prediction 2

The fact that $\frac{e^{\lambda(\kappa-1)t}-1}{\kappa-1}$ is increasing in κ follows because $\frac{\partial}{\partial\kappa}\left(\frac{e^{\lambda(\kappa-1)t}-1}{\kappa-1}\right) = g(\lambda(\kappa-1)t)/(\kappa-1)^2 \propto g(\lambda(\kappa-1)t)$ where the function $g(z) \equiv 1 + e^z(z-1)$ has derivative $e^z\, z$. Because $\kappa > 1$, $\lambda(\kappa-1)t > 0$ so that $g(\lambda(\kappa-1)t)$ is positive and increasing in κ. ■

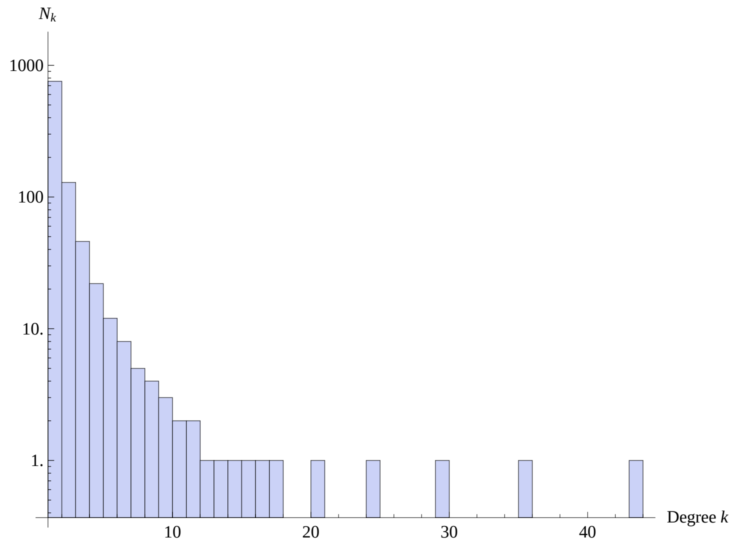
Tables

Table 1: Notation

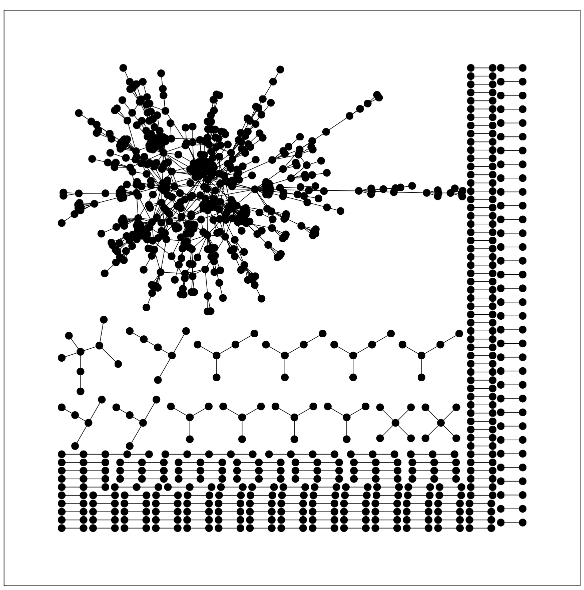
Table 1 lists notation used in this study.

Symbol	Description
N	Number of individuals in network
$k \in [1, N-1]$	Number of connections (degree) an individual has
N_k	Number of individuals with k connections (degree k)
p_k	Density of individuals with degree k; If viewed as an empirical frequency, $p_k = N_k/N$
κ	Parameter representing network heterogeneity ($\kappa \equiv \mathrm{E}[k^2]/\mathrm{E}[k] = (\sum_k k^2 p_k)/(\sum_k k p_k)$)
I_{kt}	Number of informed degree k individuals at time t ($I_{kt} \in [I_{ks}, N_k], \forall s < t$)
μ_{kt}	Portion of degree k individuals who are informed at time t ($\mu_{kt} = I_{kt}/N_k$)
μ_t	Portion of population that is informed at time t ($\mu_t = \sum_{k=1}^{N-1} p_k \mu_{kt}$)
Θ_{kt}	Density of informed neighboring individuals with degree k (i.e. an individual with degree k has average of $k\Theta_{kt}$ informed neighbors)
λ	Parameter governing speed of information flow (high λ indicates faster flow)

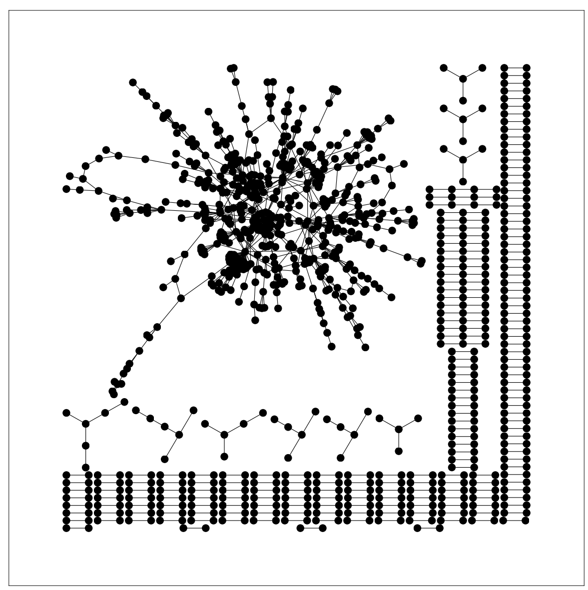
Figures



(A) Degree distribution



(B) Example network 1



(C) Example network 2

Figure 1: Example networks

Figure 1 displays an example degree distribution in Panel A for a network of 1,000 individuals where we specify the probability p_k of an individual having degree k as $p_k \propto k^{-2.25}$. The horizontal axis denotes the number of connections k an individual has and the vertical axis is the number of individuals N_k with k connections. Panels B and C display two networks generated from the degree distribution in Panel A.

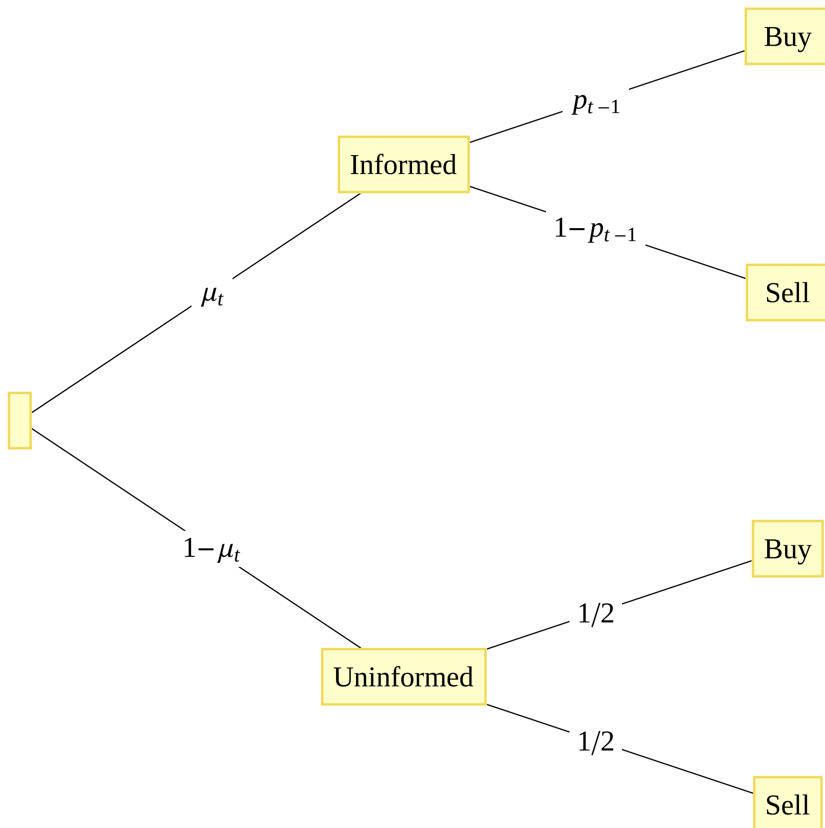
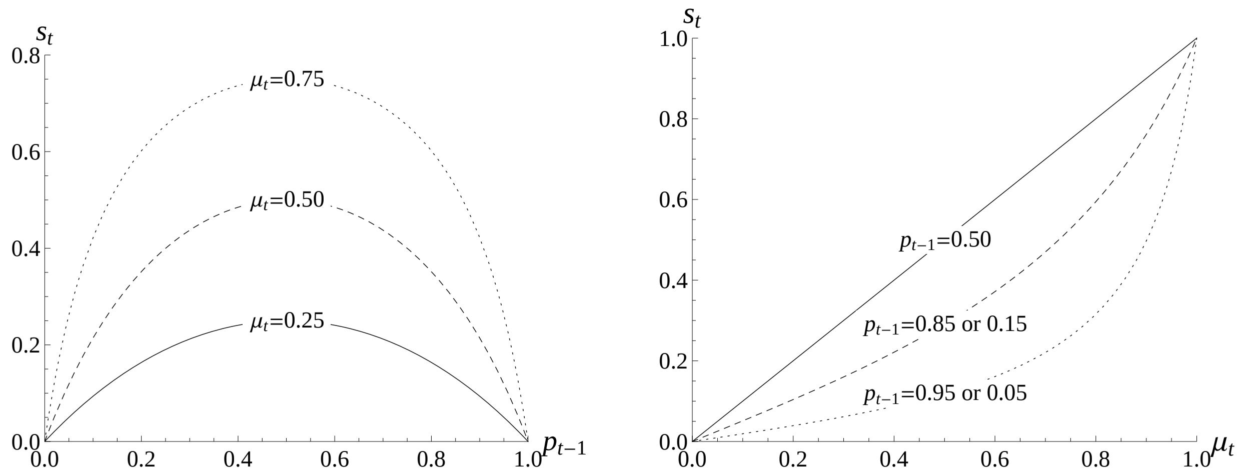


Figure 2: Probability of trades

Figure 2 displays the probabilities pertaining to trades at time t. A trader is selected at random who can trade. The trader is informed with probability μ_t, the proportion of informed traders in the population. An informed trader will buy only if the firm value is high and will sell only if the firm value is low. Prior to observing the trading choice, the market maker's prior belief that firm value is high is p_{t-1}. An uninformed trader buys and sells with equal probability.



(A) Spread as a function of prior p_{t-1} (B) Spread as a function of portion informed μ_t

Figure 3: Bid-ask spreads

Figure 3 the bid-ask spread for time t trades as a function of the market maker's prior belief p_{t-1} that firm value is high (Panel A) and as a function of the fraction μ_t of investors who are informed (Panel B). We compute the spread after normalizing the possible firm values to $\bar{v} = 1$ and $\underline{v} = 0$.

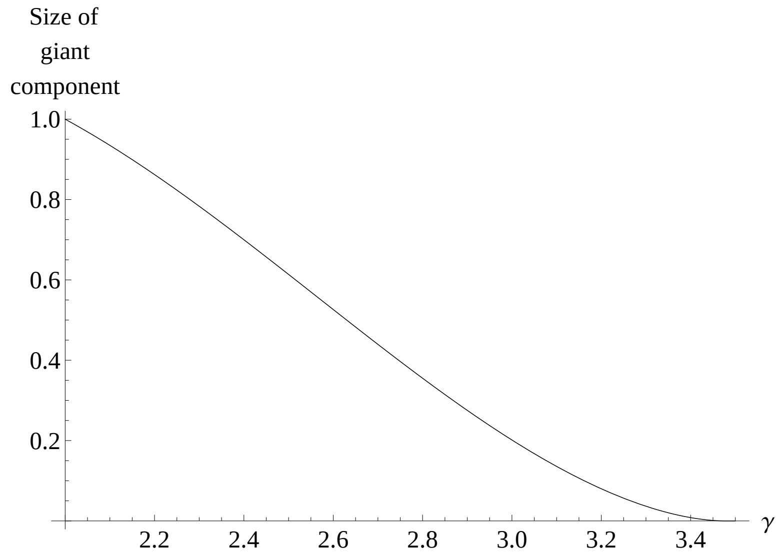
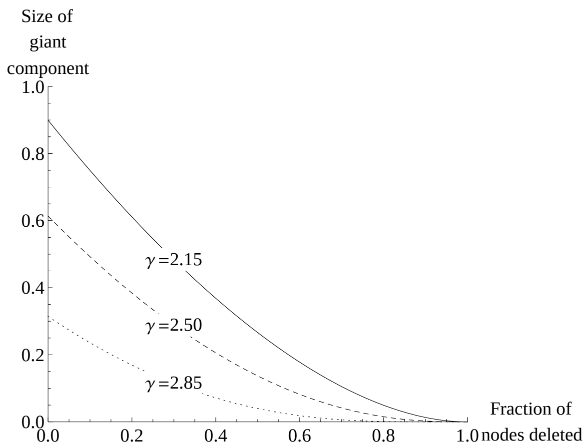
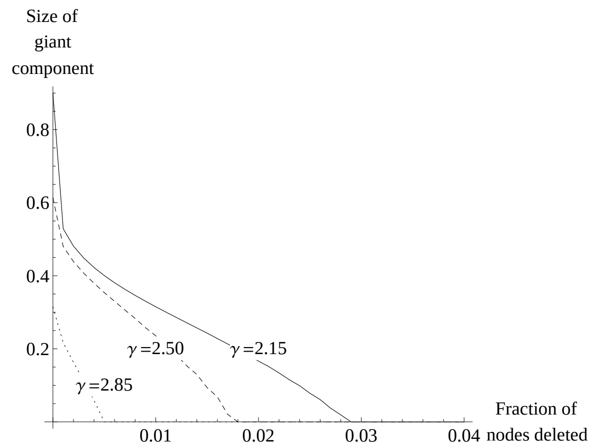


Figure 4: Size of giant component as a function of γ

Figure 4 the size of the giant component in a network with a power law degree distribution, $p_k \propto k^{-\gamma}$, as a function of the parameter γ. A giant component exists for $\gamma < 3.48$, as shown by Aiello, Chung, and Lu (2000). We determine the size of the giant component using the methods shown in Newman (2010, Section 13.8).



(A) Random deletion of nodes



(B) Targeted deletion of nodes

Figure 5: Deletion of nodes

Figure 5 displays the impact of deleting nodes on the size of the giant component in networks characterized by a power law degree distribution $p_k \propto k^{-\gamma}$. Panel A displays the effect of the random deletion of nodes. Panel B displays the effect of deleting nodes, beginning with those having the highest degree. The curves intersection with the horizontal axis corresponds to the network no longer having a giant component.