Dynamic Bonus Pools*

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Abstract

We analyze a two-period agency problem with limited liability and non-verifiable information. The principal commits to a fixed total payment which may be distributed to the agent and a third party. Unlike in a situation with unlimited liability or verifiable information, the optimal two-period contract features memory. If the agent succeeds in the first-period, second-period incentives are weakened whereas higher-powered incentives are provided if he fails.

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1 Introduction

In providing managerial incentives, firms do not rely exclusively on verifiable and objective performance information such as production quantities, accounting income, or a firm's stock price. Boards of directors (or, similarly, senior managers) often adjust these incentives based on their subjective assessments, e.g., if and to what extent market conditions influenced (favorably or unfavorably) objective performance measures. Boards also provide incentives based on their subjective assessments of a manager's cooperation, loyalty, or reputation (Murphy and Oyer 2003; Gibbs, Merchant, Van der Stede, and Vargus 2004). Given the non-verifiability of subjective assessments, boards have substantial discretion in determining bonus payments.[1] Consequently, ex post, a board may have incentives to misreport its subjective assessment in order to reduce wage payments, thus limiting the contracting usefulness of subjective signals.

Theoretical studies show that bonus-pool arrangements enable boards to use non-verifiable information when motivating managers (Baiman and Rajan 1995; MacLeod 2003). Specifically, a board commits to fund a bonus pool with a fixed payment and any subsequent discretionary bonus payment will deplete the bonus pool. Intuitively, when a board commits to a fixed payment, it has no incentives

[1]For example, consider compensation funding at UBS: "While profitability is the main factor in determining the size of our bonus pool, and while we apply funding rates that provide an initial basis for determining divisional bonus pools, management may still apply its discretion and make adjustments to further assess the overall quality of earnings by looking at relevant key performance indicators and other qualitative measures, including risk factors. Furthermore, we recognize the strategic importance of maintaining a competitive position in the labor market, and may also make adjustments to variable compensation funding determined by competitive benchmarking. . . . Such management discretion is an important element of the funding framework, enabling us to achieve a balanced outcome that considers all the relevant factors." UBS, Compensation funding and expenses, 2010 Compensation at a glance.

to misreport non-verifiable information.[2] Indeed, such bonus-pool arrangements are optimal mechanisms when a board must rely exclusively on subjective assessments to provide incentives for multiple managers (Rajan and Reichelstein 2006). This study focuses on dynamic bonus-pool arrangements, i.e., a board commits to an overall fixed payment covering multiple periods, where any non-paid amount is rolled-over to the next period.[3] Specifically, this study addresses (i), the efficiency of dynamic bonus-pool arrangements relative to single-period bonus-pool arrangements and (ii), how a dynamic bonus-pool arrangement affects the use of non-verifiable information for incentive provision.

In our study, we extend the analysis of short-term bonus-pool arrangements with a single agent (MacLeod 2003) to a two-period setting. The principal commits to fund a two-period bonus pool with a fixed payment. In each period, incentives are only provided based on subjective performance measures. The fraction of the overall bonus pool not distributed to the agent at the end of the first period is rolled over to the second period; in the second period, the principal retains discretion as to either pay out the remaining money to the agent or to partly divert it to a third party (such as a charitable organization).[4]

A first question refers to the efficiency of dynamic bonus-pool arrangements

[2]Alternatively, repeated interactions between boards and managers introduce reputational considerations that enforce relational contracts using non-verifiable information for implicit managerial incentives (Bull 1987; Baker, Gibbons, and Murphy 1994).

[3]Focusing on the wealth transfer and neglecting any incentive implications, dynamic bonus pools are applied, e.g., in settings where a firm's bonus plan limits the stock options that the firm's board can grant, over a specified time period, to executives.

[4]Following Rajan and Reichelstein (2009), a principal can also divert money to third parties if she includes an additional individual in the bonus pool arrangement who does not require incentives and whose compensation is already sufficient without any payouts from the bonus pool. Frequently, boards have the discretion to decide whether newly hired employees are entitled to participate in the allocation of an already established (dynamic) bonus pool arrangement. Accordingly, making a new employee eligible for an existing bonus-pool arrangement is tantamount to diverting funds from the bonus pool to a third party.

relative to single-period bonus-pool arrangements. Similar to studies of single-period bonus pools (Baiman and Rajan 1995; MacLeod 2003; Rajan and Reichelstein 2006, 2009; Ederhof 2010), when committing to a fixed payment in a two-period setting, a principal has no incentives to misreport non-verifiable information, thus enabling the use of non-verifiable information for allocating bonus payments.[5] We demonstrate that a dynamic bonus-pool arrangement for two periods outperforms two consecutive single-period bonus pool arrangements in terms of the principal's total wage payment for the two periods. With a dynamic bonus-pool arrangement, the principal benefits from two effects: First, with a two-period bonus pool, the principal can reduce the rent the agent earns from limited liability. Second, and more importantly, third-party payments can be reduced by pooling them in the second period.[6] While the latter result is similar to the efficiency gains from combining multiple agents in one bonus-pool arrangement (Baiman and Rajan 1995; Rajan and Reichelstein 2006, 2009; Ederhof 2010), it deviates in that it is not a different agent but rather the same agent in a future period who (partly) budget-balances with the (first-period) agent.

The second question relates to the structure of the optimal incentive contract under a dynamic bonus-pool arrangement. We find that the optimal two-period contract features memory. Specifically, if the agent obtains a positive first-period subjective assessment, the principal chooses weak second-period incen-

[5]Implicitly, our study assumes that deferring a compensation payment to the second period does not yield a profit from interest to the principal. For example, when any benefits from interest payments are included in the bonus pool or given a negligible interest rate, there is no profit from interest to the principal when she defers compensation payments to a future period.

[6]Notably, our result differs from Ederhof, Rajan, and Reichelstein (2010), Proposition 10. Different to our study, they assume that the limited liability constraints are not binding and find that the optimal two-period bonus-pool arrangement with a roll-over provision is equivalent to repeating the optimal single-period bonus-pool arrangement.

tives, whereas the principal chooses strong second-period incentives for a negative first-period subjective assessment. In the second period, given a positive first-period subjective assessment and similar to MacLeod (2003), the optimal bonus-pool arrangement pays the (remaining) bonus pool amount to the agent unless the worst possible subjective assessment materializes. Thus, the second period entails wage compression in the sense that any signal except for the worst is pooled into the same outcome to the agent. In contrast, if the agent obtains a negative first-period subjective assessment, the principal provides strong second-period incentives. Intuitively, with a negative first-period subjective assessment, no payout to the agent was made in the first period, such that the budget constraint is not binding in the second period. However, we find that the high payout only occurs for the highest subjective assessment. Consequently, the second period again entails wage compression, but now in the sense that any signal except for the best is pooled into the same outcome (i.e., a zero payment) to the agent.

Overall, similar to MacLeod (2003), the optimal dynamic bonus-pool arrangement is generally not proper in the sense that the entire bonus amount is always paid out to the participating agent. While payments to third parties are relatively unlikely if the agent obtained a positive first-period subjective assessment (i.e., only for the worst possible signal in the second period), payments to third parties are highly likely if the agent obtained a negative first-period subjective assessment (i.e., always except for the best possible signal in the second period).

Dynamic incentive problems raise also the question of how vulnerable dynamic bonus-pool arrangements are to renegotiations between contracting parties. We find that the optimal dynamic bonus pool is renegotiation-proof if the third party is also a signatory of the initial contract. For example, a publicly known policy that new employees are eligible to participate in a bonus-pool arrangement may serve as a commitment device that renders dynamic bonus-pool arrangements negotiation proof.

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 contains the analysis. After considering short-term contracts, we illustrate the benefits of long-term contracts with memory and derive the characteristics of optimal long-term contracts. Renegotiation is considered in Section 4. Section 5 addresses the setting with a risk-averse agent. Section 6 concludes.

2 Model

We consider a principal/agent-relation over two periods $t = 1, 2$. In each period, the manager (agent) provides a productive effort $a_t \in \{a^H, a^L\}$. The firm (principal) wants to implement action a^H in each period. To that purpose, in each period, the principal privately observes a subjective performance metric Y_t which can take values in $\mathcal{Y} = \{y_1, \ldots, y_n\}$.

In each period t, the probabilities $p_{it}^k = \text{Prob}\left\{Y_t = y_i \mid a^k\right\}$, $k = H, L$, satisfy the strict monotone likelihood ratio property (MLRP) $\frac{p_{it}^L}{p_{it}^H} > \frac{p_{i+1,t}^L}{p_{i+1,t}^H}$, $i = 1, ..., n-1$, i.e., higher indices indicate better news which respect to the agent having provided the desired action a^H. To simplify the statement of the results, we assume that for all realizations it holds that $p_{it}^H \neq p_{it}^L$, i.e., each observation is either good news or bad news with respect to the agent's action (for a similar assumption, see MacLeod 2003). Since in the following most contracts will take the form of binary bonus schemes it will be convenient to denote by $P_{it}^k = \sum_{j=i}^n p_{jt}^k$ the probability that a certain performance target y_i is met in period t under action k.

The principal is risk-neutral and seeks to minimize his cost of implementing $a_t = a^H$ in each period. The agent is risk-neutral. His utility is $u_1(s_1, s_2, a_1, a_2) = \sum_{t=1}^{2} s_t - c_t(a_t)$ at the beginning of the first period and $u_2(s_2, a_2) = s_2 - c_2(a_2)$ at the beginning of the second, where s_t is his compensation and $c_t(a_t)$ is his cost of effort in period t. To simplify notation, let $c_t^k = c_t(a^k)$ denote the cost of effort, $a^k, k = H, L$; for simplicity, we normalize the cost of low effort to zero, $c_t^L = 0$. The agent's outside options yield expected utilities of $U_1^R = u_1^R + u_2^R$ at the beginning of the first period and $U_2^R = u_2^R$ at the beginning of the second period.

We consider two different contracts that may be offered to the agent. Short-term contracts specify $s_{it} = s_t(y_i)$ as the payments in t if $Y_t = y_i$ is realized. Long-term contracts may exhibit memory in the sense that second-period payments $s_{ij2} = s_2(y_i, y_j)$ may depend on both Y_1 and Y_2. In both cases, the principal has to account for the fact that Y_t is subjective information which he privately observes: both the short-term and the long-tern contract cannot be enforced at a court. We therefore focus on bonus-pool contracts as analyzed by Rajan and Reichelstein (2009), where the principal commits to a constant total payment $\overline{s}$ which he may distribute among the agent and a third party, such as a charity. The total payment is observable and thus enforceable and, ex post, the principal has no incentive to misreport the agent's performance. Given a short-term contract, a bonus pool $\overline{s}_t$ is set up for each period, whereas with a long-term contract a bonus pool $\overline{s}$ is specified for both periods.

Due to the non-verifiable performance measure, an incentive problem arises even under the assumption of a risk-neutral agent. We assume that the firm cannot be sold to the manager because the latter is of restricted wealth. More formally, we assume that payments have to exceed a minimum level s^{min} in each period. Limited liability implies an incentive problem even with a risk-neutral agent, which has already been analyzed in models with verifiable information (e.g. Innes 1990, Kim 1995 and Demougin and Fluet 1998). In this sense, we combine in our model the contracting frictions arising from limited liability and unverifiable information.

3 Analysis

3.1 Short-term contracts

If short-term contracts are applied, the principal's problem to minimize his payment $\overline{s}_t$ in period t is similar to that analyzed by MacLeod (2003), except that we take into account the agent's liability level s^{min}. For period $t = 1, 2$, the cost minimization program takes the form

$$\min_{s_1,\dots s_n} \overline{s}_t \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} p_{it}^H s_{it} - c_t^H \geq u_t^R \tag{2}$$

$$\sum_{i=1}^{n} p_{it}^H s_{it} - c_t^H \geq \sum_{i=1}^{n} p_{it}^L s_{it} \tag{3}$$

$$s_{it} \leq \overline{s}_t \tag{4}$$

$$s_{it} \geq s^{min}. \tag{5}$$

The principal's objective is to minimize his fixed payment $\overline{s}_t$. The individual rationality constraint, (2), ensures that the agent will sign the contract and the incentive compatibility constraint, (3), guarantees that the agent will choose a^H. Constraint (4) is the bonus-pool constraint, stating that the payments to the agent must not exceed the size of the bonus pool. The liability constraints, (5), require all payments to be at least equal to s^{min}.

If the liability constraints, (5), do not bind, the optimal contract takes the form derived in MacLeod (2003): it is a binary contract in which a bonus is paid for all but the poorest performance. Only if y_1 is realized, the amount is paid to the third party. This contract is designed such that payments to the third party are minimized because third-party payments are lost for the provision of expected utility to the agent. With the agent earning a rent, however, this is no longer important, i.e., only the absolute bonus amount matters, but not its expectation. The optimal contract under limited liability therefore significantly differs from the optimal contract derived by MacLeod (2003).[7]

Proposition 1 *If in the optimal short-term contract the liability constraint (5) is binding such that the agent earns a rent from limited liability, this contract is binary and takes the form*

$$s_{it} = \begin{cases} s^{min} & \text{if} \quad p_{it}^H < p_{it}^L \\ s^{min} + \frac{c_t^H}{P_{t+}^H - P_{t+}^L} & \text{else.} \end{cases} \tag{6}$$

where $P_{t+}^k = \sum_{\{i|p_{it}^H > p_{it}^L\}} p_{it}^k$ *is the probability of outcomes which are more likely under* a^H.

Proposition 1 shows that the absolute bonus which is necessary to induce the high effort level is minimized by refusing its payment in all cases which are more

[7] All proofs are included in the Appendix.

likely under a^L than under a^H, thus bringing bad news with respect to the agent's action. For illustration, rearrangement of the incentive compatibility constraint, (3), yields

$$\sum_{i=1}^{n} \left[p_{it}^H - p_{it}^L\right] s_{it} \geq c_t^H.$$

If $p_{it}^H < p_{it}^L$, the term in brackets is negative, and the incentive problem is aggravated by any positive payment s_{it}. Therefore, the principal does best by paying the minimum wage s^{min} in all of these instances.

The distinction between good news and bad news will play a central role in the subsequent analysis of the long-term contract. Therefore, it will be convenient to denote $I_t^- = \{i \mid p_{it}^H < p_{it}^L\}$ as the set of realizations which convey bad news in period t, and $I_t^+ = \{i \mid p_{it}^H \geq p_{it}^L\}$ as the set of realizations with good news. We also state that for I_t^- the agent has "failed" in the first period, whereas for I_t^+ the agent has "succeeded" in the first period.

If there is only one realization for which $p_{it}^H < p_{it}^L$, the structure of the contract under unlimited liability and limited liability obviously are the same. Contrary, if I_t^+ contains only one element, the optimal bonus contract under limited liability yields payments identical to those in an contract with verifiable information and limited liability, where a bonus is paid only for the best performance, y_n (see Demougin and Fluet 1998). Since we are interested in the effects of limited liability and non-verifiable information, we preclude these results by the following assumption:

Assumption 1 *In each period, both sets I_t^- and I_t^+ have at least two elements.*

The following example illustrates the difference between the three cases mentioned above:

Example 1 *Let the probabilities in period $t = 1, 2$ be given by*

p_{it}^k	y_1	y_2	y_3	y_4
a^L	0.4	0.3	0.2	0.1
a^H	0.1	0.2	0.3	0.4

Thus, y_1 and y_2 convey bad news about the agent's action, whereas y_3 and y_4 are good news. The agent's cost of high effort is $c_t^H = 1$, the minimum wage is $s^{min} = 0$, and the agent's reservation level of utility u_t^R may take values 0 or 2.

Case 1: Non-verifiable information. With $u_t^R = 2$, the liability constraint (3) is not binding, and the optimal bonus pool contract is

$$s_{it} = \begin{cases} 1 & \textit{for} \quad i = 1 \\ \frac{13}{3} & \textit{for} \quad i = 2, 3, 4 \end{cases}$$

and has the structure as in McLeod (2003).

Case 2: Non-verifiable information and limited liability. With $u_t^R = 0$, the liability constraint, (3), is binding, and the optimal bonus-pool contract is

$$s_{it} = \begin{cases} 0 & \textit{for} \quad i = 1, 2 \\ \frac{5}{2} & \textit{for} \quad i = 3, 4. \end{cases}$$

Case 3: Verifiable information. In contrast, a contract

$$s_{it} = \begin{cases} 0 & \textit{for} \quad i = 1, 2, 3 \\ \frac{10}{3} & \textit{for} \quad i = 4 \end{cases}$$

of the form derived by Demougin and Fluet (1998) would be offered if Y_t were contractible information.

In Case 1, the agent exactly achieves his reservation level u_t^R of expected utility. In Case 2, the agent earns a rent from limited liability. The required bonus for good performance is lower in Case 2 of limited liability relative to Case 1 without limited liability (5/2-0=2.5 instead of 13/3-1=3.33). But if a contract with that bonus agreement (bonus 2.5 if i=3,4) were offered in Case 1 with non-binding liability constraint, the principal would have to increase the base salary for y_1 and y_2 from 1 to 2.25 to make the agent sign the contract, incresing total compensation cost to 2.25+2.5=4.75. Therefore, the principal benefits from paying the higher bonus 3.33 with a higher probability (.9 instead of .7) to keep third-party payments to a minimum and provide the agent with the highest possible expected utility that is possible without completely destroying incentives.

Comparing the bonuses in Cases 2 and 3, we observe that the bonus is also higher in a situation with verifiable performance information.[8] But since this bonus is only paid with probability $.4$, whereas the payment 2.5 in Case 2 is due in any instance (either to the agent or a third party), the principal's compensation costs are lower with verifiable information relative to the setting with non-verifiable information ($.4 \cdot 3.33 = 1.33$ instead of 2.5).

3.2 Long-term contracts with memory

In the long-term contract, the principal may offer different contracts in the second period, depending on the realization of the agent's first-period performance. The

[8]In fact, the bonus is identical to that in the first contract. But this identity is owed to the symmetry of the example, where $p_{1t}^L - p_{1t}^H = p_{4t}^H - p_{4t}^L$.

principal's cost-minimization problem then takes the form

$$\min_{\{s_{i1}\},\{s_{ij2}\}} \quad \overline{s} \tag{7}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} s_{i1} p_{i1}^{H} - c_1^H + \sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij2} p_{i1}^{H} p_{j2}^{H} - c_2^H \geq U_1^R \tag{8}$$

$$\sum_{j=1}^{n} s_{ij2} p_{j2}^{H} - c_2^H \geq U_2^R \quad \forall\, i \tag{9}$$

$$\sum_{i=1}^{n} s_{i1} p_{i1}^{H} - c_1^H + \sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij2} p_{i1}^{H} p_{j2}^{H} - c_2^H$$
$$\geq \sum_{i=1}^{n} s_{i1} p_{i1}^{L} + \sum_{i=1}^{n}\sum_{j=1}^{n} s_{ij2} p_{i1}^{L} p_{j2}^{H} - c_2^H \tag{10}$$

$$\sum_{j=1}^{n} s_{ij2} p_{j2}^{H} - c_2^H \geq \sum_{j=1}^{n} s_{ij2} p_{j2}^{L} \quad \forall\, i \tag{11}$$

$$s_{i1} + s_{ij2} \leq \overline{s} \quad \forall\, i, j \tag{12}$$

$$s_{i1} \geq s^{min}, s_{ij2} \geq s^{min} \quad \forall\, i, j \tag{13}$$

The principal wants to minimize the total payment $\overline{s}$ of both periods. Inequalities (8) and (9) state the agent's individual rationality constraints at the beginning of the first and second period. Constraint (9) is a set of second-period individual rationality constraints which are contingent on the agent's first-period performance. This contingency is due to the fact that the principal may offer different contracts for different first-period performances. For all of these contracts, the agent's expected utility has to be at least equal to his second-period outside option U_2^R.

Contingencies are also present in the incentive compatibility constraints, (10) and (11). The second-period constraints (11) depend the agent's first-period performance. For all outcomes, the agent has to prefer a^H to a^L. For the first period, the incentive compatibility constraint, (10), is more subtle because it entails the second-period consequences of the agent's first-period action. To let the agent prefer $a_1 = a^H$ over $a_1 = a^L$, his direct benefits from expected first-period performance plus the indirect benefits from potentially more attractive second-period contracts have to exceed the cost c_1^H of high effort in the first period. Since (11) guarantees that the agent will choose a^H in the second period, he will anticipate $a_2 = a^H$ when calculating these second-period benefits.

Wages s_{i1} and s_{ij2} are related to the principal's objective function by the bonus pool constraints, (12), stating that the total payment of both periods must not exceed the bonus pool size $\overline{s}$ for any realization of (Y_1, Y_2). Finally, the liability constraints,(13), ensure that no payment falls short of the minimum wage s^{min}.

In a first step, we show that postponement of payments can be used to improve short-term contracts. In the subsequent section, we characterize the optimal contract.

Proposition 2 *If in the optimal short-term contract the liability constraint (5) is binding such that the agent earns a rent from limited liability, then in the optimal long-term contract, second-period payments will nontrivially depend on first-period outcomes, i.e., the optimal long-term contract has memory.*

In a long-term contract, the principal can reduce compensation cost by providing first-period incentives via the prospect of a more attractive second-period contract under first-period success. To that purpose, he may extend the set of realizations where the agent is entitled to receive a bonus payment in the second period, provided he succeeded in the first period. This helps to cut third-party payments and increases the agent's second-period utility. Since the modified contract is only offered under I_1^+, the utility increase works as an additional first-period incentive, and the first-period bonus can be reduced. Obviously, this reduction comes at a cost because as we have already seen in subsection 3.1, the second-period bonus has to be increased under lower-powered incentives in order to implement the desired effort level. Proposition 2 essentially shows that these costs are outweighed by the first-period savings, implying that a long-term contract with memory is beneficial to the principal.

The procedure is illustrated in the following extension of the example:

Example 1 *(cont.) Suppose the data of the example above applies in both periods. The short-term contract*

$$s_{it} = \begin{cases} 0 & \textit{if}\, i = 1, 2 \\ \frac{5}{2} & \textit{else} \end{cases}$$

yields a total compensation cost of $2 \cdot \frac{5}{2} = 5$. *This contract can be improved in the above described manner: In the second period, the agent receives*

$$s_{ij2} = \begin{cases} 0 & \textit{if}\, j = 1, 2 \\ \frac{5}{2} & \textit{else} \end{cases}$$

in case that $Y_1 \in \{y_1, y_2\}$ *and*

$$\hat{s}_{ij2} = \begin{cases} 0 & \textit{if}\, j = 1 \\ \frac{10}{3} & \textit{else} \end{cases}$$

if $Y_1 \in \{y_3, y_4\}$ *is realized in the first period. Thus, under* I_1^+*, he receives a bonus of* $\frac{10}{3}$ *with probability 0.9, whereas under* I_1^-*, he receives a bonus of* $\frac{5}{2}$ *with probability 0.7. The difference in expected utility is* $\Delta_2 = \frac{9}{10} \cdot \frac{10}{3} - \frac{7}{10} \cdot \frac{5}{2} = 3 - \frac{7}{4} = \frac{5}{4}$*. Thus, the bonus for the first period can be reduced by* $\frac{5}{4}$*, which yields*

$$s_{i1} = \begin{cases} 0 & \text{if } i = 1, 2 \\ \frac{5}{2} - \frac{5}{4} = \frac{5}{4} & \text{else.} \end{cases}$$

Total compensation cost is $s = \frac{5}{4} + \frac{10}{3} = \frac{55}{12} < 5$*.*

Note that the described improvement is only possible if both limited liability and unverifiable information are present. Without limited liability, only for the worst performance no bonus is paid in the short-term contract. Therefore, extending the set of outcomes for which a bonus is paid would destroy incentives. With verifiable information and limited liability, the same procedure could well be applied. However, all payments will only be due for the principal with a certain probability, implying that the costs and benefits of the variation exactly balance.

3.3 Optimal long-term contract

The last subsection proved that improvements are possible by introducing memory in the contract. This was done by changing the second-period contract for good news in the first period, i.e., the principal provides first-period incentives by offering a different second-period contract. This effect can be supplemented by changing the contract for bad news in the first period, too. Obviously, this has to be done in the opposite direction, i.e., by restricting the realization of Y_2 for which a bonus is paid. To maintain second-period incentives, the bonus in the second period has to be increased. With respect to total compensation cost, such an increase for small variations has no effect because we consider those realizations of first-period performance in which no bonus was paid. The bonus pool therefore has some leeway to extend the second-period bonus.

To what extent these two instruments are used in the optimal contract depends on the parameters of the problem. In our example, it looks like follows:

Example 1 *(cont.) Suppose again that the initial data hold for both periods. The optimal long-term contract takes the form*

$$s_{i1} = \begin{cases} 0 & \textit{if } i = 1,2 \\ \frac{5}{6} & \textit{if } i = 3,4 \end{cases}$$

in the first period, and in the second period the agent receives

$$s_{ij2} = \begin{cases} 0 & \textit{if } j = 1,2,3 \\ \frac{10}{3} & \textit{else} \end{cases}$$

in case that $Y_1 \in \{y_1, y_2\}$ *and*

$$\hat{s}_{ij2} = \begin{cases} 0 & \textit{if } j = 1 \\ \frac{10}{3} & \textit{else} \end{cases}$$

if $y_i \in \{y_3, y_4\}$*. The principal's compensation cost is* $\overline{s} = \frac{5}{6} + \frac{10}{3} = \frac{25}{6}$.

Two issues are noteworthy. The first issue relates to the origin of the principal's benefits from the two-period contract, as compared to a repeated short-term contract in each period. These benefits not only arise from a reduction of agent's rent from limited liability, but also from a reduction of the third-party payment. In the example, with two consecutive short-term contracts, the agent's ex-ante expected utility is $2 \cdot \left[\frac{7}{10} \cdot \frac{5}{2} - 1\right] = \frac{3}{2}$, whereas the third party receives $2 \cdot \frac{3}{10}\frac{5}{2} = \frac{3}{2}$ in expectation. Under the two-period contract, the agent's expected utility is $\frac{3}{10}\left[0 + \frac{2}{5}\frac{10}{3}\right] + \frac{7}{10}\left[\frac{5}{6} + \frac{9}{10}\frac{10}{3}\right] - 2 = \frac{13}{12}$ while the expected third-party payment is $\frac{3}{10}\left[\frac{5}{6} + \frac{3}{5}\frac{10}{3}\right] + \frac{7}{10}\left[\frac{1}{10}\frac{10}{3}\right] - 2 = \frac{13}{12}$. Thus, the principal's cost savings of $5 - \frac{25}{6} = \frac{5}{6}$ consists of a rent reduction and a reduced third-party payment of $\frac{3}{2} - \frac{13}{12}$ each.[9]

The second issue relates to the structure of the contract. In the second period, it has an extreme form. If the agent has "failed" in the first period (i.e., for I_1^-), no bonus was due and the budget constraint of the bonus pool is no longer binding. The offered contract therefore has the form as with verifiable information, offering a high bonus, but only for the best performance. If the agent "succeeded" in the first period (i.e., for I_1^+), the logic is reversed: now the budget constraint is binding, but the economics of limited liability are mitigated by the fact that first-period incentives are provided by second-period bonuses. The contract therefore has the structure as under unlimited liability and unverifiable information, paying a bonus for all but the worst performance.

This extreme structure will only be observed in special cases, where the rents produced by limited liability are high enough in both periods. In general, the contract may be modified in different extent. The general structure, however, will

[9]The identity of the two amounts is due to the symmetry of the example. Differing amounts may occur, but the benefits in general will arise from both origins.

remain. In the following proposition, we describe its most important features.

Proposition 3 *If the agent earns a rent from limited liability in the optimal long-term contract, this contract has the following properties:*

1. *The first-period compensation has the form*

$$s_{i1} = \begin{cases} s^{min} & \textit{if } i \in I_1^- \\ s_i \geq s^{min} & \textit{if } i \in I_1^+ \end{cases} \tag{14}$$

2. *If $s_i > s^{min}$ for some $i \in I_1^+$ in the first period, the second-period compensation for i has the form:*

$$s_{ij2} = \begin{cases} s_{i12} \geq s^{min} & \textit{if } j = 1 \\ s_{i12} + \frac{c_2^H}{p_{12}^L - p_{12}^H} & \textit{else} \end{cases} \tag{15}$$

Proposition 3 states that in the first period the mere structure of the limited liability contract will remain: Only the minimum payments will be made if the agent's performance conveys bad news. With good news, however, the bonus can be postponed (partly or completely) to the second period. More importantly, Proposition 3 states that if first-period incentives cannot completely be provided by second-period payments, the second period incentive contract will take the form proposed by MacLeod (2003) and Rajan and Reichelstein (2009) for the case without binding liability constraints, i.e., a bonus is refused only for the worst performance. Following the argument in Section 3.1, in the second period, the agent thus obtains the maximum expected utility for given incentives. Therefore, the described contract makes use of the cost-reducing modifications characterized in Proposition 2 to the full extent, provinding first-period by an improved second-period contract under good news.[10]

It is instructive to ask how our results relate to Ederhof et al. (2010), Proposition 10. Specifically, the fact that the principal's benefit from a dynamic contract arise not only from a reduction of the agent's rent provokes the question why similar savings are not possible if the agent does not earn a rent from limited liability. In a model with a binary performance measure and a risk-averse agent with unlimited wealth, Ederhof et al. (2010), Proposition 10, show that the optimal two-period contract is equivalent to a repetition of the optimal one-period contract. The difference between their result and ours can be explained by the liability restriction imposed in our model: if the agent does not earn a rent from limited liability in the one-period contract, none of the two contract modifications described above is feasible and beneficial. Under I_1^-, the contract modification is not possible because it results in a reduction of the agent's second-period utility, which is impossible if he does not earn a rent. In turn, under I_1^+, the proposed modification requires an extension of the bonus payment to outcomes with poorer performance. But since in the optimal short-term contract without limited liability the bonus is refused only for the poorest performance, such an extension is not beneficial to the principal.[11]

[10]With the contract characterized in Proposition 3, the principal implements a^H in both periods. Moreover, given that the bonus pool is fixed, the principal has no strict incentives to strategically deviate from truthfully reporting the observed performance.

[11]Additionally, Ederhof et al. (2010) consider a setting with a binary performance measure, rendering the contract modifications suggested in Proposition 3 impossible. However, their result would also hold in a framework with more outcomes as long as the agent's participation constraint is binding. A more detailed comparison is provided in Section 5

4 Renegotiation

Considering renegotiation forces us to think in more detail about the nature of the contract including payments to a third party. The most important question in this respect is whether the third party is a signatory of this contract or benefits from the contract in a passive way - as a third party in a narrower sense.

Following the principle of "those who make a contract, may unmake it" (Beatty v. Guggenheim Exploration Co., 122 N.E. 378, 380 (N.Y. 1919)) also in its negative sense, renegotiation is not possible without the third party if it has signed the initial contract, even if it has no duties from this contract. In this case, any second-period contract which implements the desired action a^H is efficient if the third party is risk neutral. Under risk neutrality, for a given action, the distribution of money is a zero-sum game. Any change of the contract may only yield an inefficient action a^L, reducing the pie to be shared, or lead to a distribution of outcome that makes at least one party worse off. The parties therefore never will agree upon a change, and the contract presented in Section 3.3 is renegotiation proof.

If the third party is not a signee, the contract presented in the preceding section is not renegotiation proof. After the first period has elapsed, the principal and the agent will have an incentive to change the contract to that incentive compatible one which minimizes the payment to the third party because this way the pie to be shared among the two is maximized. Since this contract is the same for all realizations of Y_1, the provision of first-period incentives by offering different second-period contracts under good news and bad news would be completely removed by such renegotiation: in order to maximize his part of the surplus, the principal will have an incentive to always report bad news in the first period and not to pay the promised bonus, even if he has observed good news.

5 Risk-averse agent

The contrast between Proposition 2 and the result of Ederhof and al. (2010) raises the question of what mainly drives our result. As we have already argued in Section 3.3, limited liability is crucial because the resulting rents the agent earns in a one-period contract give the opportunity to offer second-period contracts of different expected utility to the agent, thereby providing first-period incentives. We now analyze whether this kind of contract modification is also beneficial if the agent is risk-averse.

With a risk-averse agent, two aspects could hinder such benefits. First, the contract modification characterized in Proposition 2 exchanges part of the first-period bonus against an increased second-period bonus. Since the second-period bonus is only due if the agent succeeds, this introduces additional uncertainty into the contract. With a risk-averse agent, a higher risk premium will result. From the outset it is not clear whether the benefits from the contract modification outweigh these costs. Second, risk aversion may also give rise to the issue of consumption smoothing over periods. As can be seen from the example, compensation varies substantially in the modified contract, even if the agent succeeds in both periods.

Subsequently, we will show that none of these aspects affects the validity of our main result. To that purpose, we consider the extreme situation where the agent's utility is completely independent over periods, without an opportunity to smooth consumption via the capital market. Intuitively, this assumption should favor contracts which are similar over time. Formally, the agent's utility is given by $u_1(s_1, s_2, a_1, a_1) = \sum_{t=1}^{2} v(s_t) - c_t(a_t)$ at the beginning of the first period and $u_2(s_2, a_2) = v(s_2) - c_2(a_2)$ at the beginning of the second, where $v(\cdot)$ denotes the agent's utility from wealth and $v' > 0$ and $v'' < 0$ imply the agent's strict risk aversion. All other model assumption remain valid.

In a first step, we analyze the optimal one-period contract. The principal's optimization problem becomes

$$\min_{s_1,\dots s_n} \bar{s}_t \tag{16}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} p_{it}^H v(s_{it}) - c_t^H \geq u_t^R \tag{17}$$

$$\sum_{i=1}^{n} p_{it}^H v(s_{it}) - c_t^H \geq \sum_{i=1}^{n} p_{it}^L \, v(s_{it}) \tag{18}$$

$$s_{it} \leq \bar{s}_t \tag{19}$$

$$s_i \geq s^{min}. \tag{20}$$

If the liability constraint, (20), is not binding, the contract has the structure proposed by MacLeod (2003), as it was already described in Section 3.1. If the agent earns a rent from limited liability, the optimal short-term contract has the same structure as under risk neutrality, stipulating a bonus for good news:

Proposition 4 *If in the optimal short-term contract with a risk-averse agent the liability constraint (5) is binding such that the agent earns a rent from limited liability, this contract is binary and takes the form*

$$s_{it} = \begin{cases} s^{min} & \text{if} \quad p_{it}^H < p_{it}^L \\ v^{-1}\left(v(s^{min}) + \frac{c_t^H}{P_{t+}^H - P_{t+}^L}\right) & \text{else}, \end{cases} \tag{21}$$

where $P_{t+}^k = \sum_{\{i|p_{it}^H > p_{it}^L\}} p_{it}^k$.

Risk sharing issues are of no relevance in the optimal contract because, following limited liability, the agent's expected utility exceeds his reservation utility. Therefore, the same structure of the optimal contract as under risk neutrality results.

This structure, in turn, affords the same contract modifications as proposed in Subsections 3.2 and 3.3 for a risk-neutral agent. As a consequence, the optimal two-period contract has memory also for a risk-averse agent.

Proposition 5 *If in the optimal short-term contract for a risk-averse agent the liability constraint (20) is binding such that the agent earns a rent from limited liability, then in the optimal long-term contract, second-period payments will non-trivially depend on first-period outcomes, i.e., the optimal long-term contract has memory.*

The proof to Proposition 5 demonstrates that at least under first-period bad news a modification of the second-period compensation is also beneficial with a risk-averse agent. It therefore ties in with our description in Section 3.2 of how short-term contracts can be improved. Since in the case of bad news no bonus is paid in the first period, there is some leeway for higher-powered incentives in the second period, leading to a lower expected utility from second-period compensation for the agent. Since no such utility loss is realized under first-period good news, the agent experiences an additional punishment for failure that –due to his limited liability– cannot be given by direct first-period wage cuts. Dynamic bonus pools agreements therefore in some sense help to break the liability constraint of the first period. This can be used to reduce the first-period bonus, thereby decreasing the size of the overall bonus pool.

6 Conclusion

Our analysis shows that bonus pools significantly change if they are applied in a multi-period agency setting. By postponing first-period bonus payments to the second period, the principal may not only reduce the agent's rent from limited liability, he may also save payments which would otherwise be transferred to a third party. First-period incentives are provided by offering different second-period contracts under good news and bad news in the first period: If first-period performance provides good news, the second-period contract gives low-powered incentives, offering a high expected utility to the agent. Under bad news for the agent's first-period performance, second-period incentives are high-powered, providing a low expected utility to the agent. This procedure is well in line with corporate practice, where low performers frequently are given a second chance, but with more demanding targets to be met. To prevent renegotiation of such a contract, it is important to involve the third party as a signee into the bonus-pool arrangement.

Appendix - Proof of Propositions

Proof of Proposition 1:

From the Lagrangian of the problem (1) – (5), the first order condition

$$\frac{\partial L}{\partial s_{it}} = \lambda_t p_{it}^H + \mu_t \left[p_{it}^H - p_{it}^L\right] - \nu_{it} + \eta_{it} = 0$$

can be derived, where $\lambda_t, \mu_t, \{\nu_{it}\}_{i=1,\ldots,n}$ and $\{\eta_{it}\}_{i=1,\ldots,n}$ are the multipliers of the constraints (2), (3), (4) and (5). If the agent earns a rent, the participation constraint (2) is not binding and therefore $\lambda_t = 0$. Furthermore, $\mu_t > 0$ must hold because otherwise the incentive constraint would not bind. Then, however, the principal could save payments by decreasing all payments larger than s^{min} by some small amount, thus reducing the bonus pool size $\overline{s}_t$. Therefore, since both ν_{it} and η_{it} are nonnegative by definition, it must hold that $\nu_{it} > 0$ if the term in brackets is positive and $\eta_{it} > 0$ if the term in brackets is negative. Hence, if $p_{it}^H > p_{it}^L$, the budget constraint (4) will be binding, whereas for $p_{it}^H < p_{it}^L$ the liability constraint (5) will be binding. From these facts the binary structure of the contract follows.

Under the binary structure, the incentive constraint (3) can be re-stated as

$$s^{min} + P_{t+}^H \left[\overline{s}_t - s^{min}\right] - c_t^H \geq s^{min} + P_{t+}^L \left[\overline{s}_t - s^{min}\right]$$

or

$$\left[P_{t+}^H - P_{t+}^L\right] \left[\overline{s}_t - s^{min}\right] \geq c_t^H$$

from which the cost-minimizing size of the bonus pool,

$$\overline{s}_t = s^{min} + \frac{c_t^H}{P_{t+}^H - P_{t+}^L},$$

follows. □

To prove Proposition 2, it is helpful to first consider the following lemma:

Lemma 1 *If the likelihood ratio p_i^L/p_i^H of probability functions p_i^k is strictly decreasing in i, this likelihood ratio for $i > 1$ is strictly smaller than the likelihood ratio F_i^L/F_i^H of the corresponding cumulative distribution functions $F_i^k = \sum_{j=1}^{i} p_j^k$, i.e,*

$$p_i^L/p_i^H < F_i^L/F_i^H \quad \forall\, i > 1.$$

Proof Since $p_1^k = F_1^k$, it suffices to show that from $p_{i-1}^L/p_{i-1}^H \leq F_{i-1}^L/F_{i-1}^H$ and strict MLRP it follows that $p_i^L/p_i^H < F_i^L/F_i^H$. Strict MLRP and $p_{i-1}^L/p_{i-1}^H \geq F_{i-1}^L/F_{i-1}^H$ imply that

$$\frac{p_i^L}{p_i^H} < \frac{F_{i-1}^L}{F_{i-1}^H} \quad \text{or} \quad \frac{F_{i-1}^H}{p_i^H} < \frac{F_{i-1}^L}{p_i^L}.$$

Adding 1 on both sides yields

$$\frac{p_i^H + F_{i-1}^H}{p_i^H} < \frac{p_i^L + F_{i-1}^L}{p_i^L}$$

which due to the fact that $p_i^k + F_{i-1}^k = P_i^k$ gives

$$\frac{F_i^H}{p_i^H} < \frac{F_i^L}{p_i^L} \quad \Leftrightarrow \quad \frac{p_i^L}{p_i^H} < \frac{F_i^L}{F_i^H}.$$

□ □

Proof of Proposition 2:

The proof of Proposition 2 is by construction. Suppose short-term contracts are used in both periods. We show that the principal can do better. To that purpose, he may change the second-period contract for those realizations of Y_1 where $p_{i1}^H \geq p_{i1}^L$, i.e. where a bonus is due in the short-term contract. Instead of paying

$$s_{ij2} = \begin{cases} s^{min} & \text{if } j \in I_2^- \\ s^{min} + \frac{c_2^H}{P_{2+}^H - P_{2+}^L} & \text{if } j \in I_2^+ \end{cases}$$

if $i \in I_1^+$, where $P_{2+}^k = \sum_{I_2^+} p_{i2}^k$, as in the optimal short-term contract, the principal offers $s_{ij2} + \Delta s$ where the contract variation

$$\Delta s = \begin{cases} \frac{\delta}{p_{j2}^L - p_{j2}^H} & \text{if } j = \hat{j} = \max I_2^- \\ \frac{\delta}{P_{2+}^H - P_{2+}^L} & \text{if} j \in I_2^+ \\ 0 & \text{else} \end{cases}$$

is constructed such that the incentive compatibility constraint (11) still holds with equality, and $\delta > 0$ is some small amount.

The principal extends bonus payments to the highest realization $\hat{j} = \max I_2^-$ of Y_2 which is more likely under a^L. This extension decreases incentives, and the bonus for I_2^+ has to be increased. The principal's second period compensation cost therefore increases by $\Delta_2 = \delta/(P_{2+}^H - P_{2+}^L)$. At the same time, however, the agents expected second-period utility increases by

$$\Delta_1 = p^H_{\hat{j}2} \frac{\delta}{p^L_{\hat{j}2} - p^H_{\hat{j}2}} + P^H_{2+} \frac{\delta}{P^L_{2+} - P^H_{2+}}.$$

The agent will anticipate this increase. Thus, to keep first-period incentives, his first-period compensation for $i \in I_1^+$ may be decreased by Δ_1. Since a positive bonus is paid for $i \in I_1^+$ in the short-term contract, such a decrease is possible for small δ and decreases the principal's first period compensation cost by Δ_1.

In total, the principal's compensation cost is lower under the variation if the saving Δ_1 in the first period exceeds the cost Δ_2 in the second. This is the case if

$$\frac{p^H_{\hat{j}2}}{p^L_{\hat{j}2} - p^H_{\hat{j}2}} + \frac{P^H_{2+}}{P^H_{2+} - P^L_{2+}} > \frac{1}{P^H_{2+} - P^L_{2+}}$$

or

$$\frac{p^H_{\hat{j}2}}{p^L_{\hat{j}2} - p^H_{\hat{j}2}} > \frac{1 - P^H_{2+}}{P^H_{2+} - P^L_{2+}}.$$

Taking into account that $1 - P^k_{2+} = F^k_{\hat{j}2}$ and $P^H_{2+} - P^L_{2+} = F^L_{\hat{j}2} - F^H_{\hat{j}2}$, where $F^k_{it} = \sum_{j=1}^{i} p^k_{it}$ is the cumulative distribution function, denoting the probability that Y_t does not exceed y_i under action k, this can be written as

$$\frac{p^H_{\hat{j}2}}{p^L_{\hat{j}2} - p^H_{\hat{j}2}} > \frac{F^H_{\hat{j}2}}{F^L_{\hat{j}2} - F^H_{\hat{j}2}} \quad \Leftrightarrow \quad \frac{p^L_{\hat{j}2} - p^H_{\hat{j}2}}{p^H_{\hat{j}2}} < \frac{F^L_{\hat{j}2} - F^H_{\hat{j}2}}{F^H_{\hat{j}2}} \quad \Leftrightarrow \quad \frac{p^L_{\hat{j}2}}{p^H_{\hat{j}2}} < \frac{F^L_{\hat{j}2}}{F^H_{\hat{j}2}}$$

which is always fulfilled under MLRP (see Lemma 1). □

Proof of Proposition 3:

To analyze the principal's optimization problem (7) - (13), let λ_1 and $\{\lambda_{i2}\}_{i=1,\dots,n}$ denote the multipliers of the individual rationality constraints (8) and (9), μ_1 and

$\{\mu_{i2}\}_{i=1,\ldots,n}$ be those of the incentive compatibility constraints (10) and (11), $\{\nu_{ij}\}_{i=1,\ldots,n,j=1,\ldots,n}$ be the multipliers of the budget constraints (12), and $\{\eta_i\}_{i=1,\ldots,n}$ and $\{\eta_{ij}\}_{i=1,\ldots,n,j=1,\ldots,n}$ be those of the first-period and second-period liability constraints (13).

To prove claim 1, consider the first-order condition with respect to the first-period compensation s_{i1},

$$\frac{\partial \mathcal{L}}{\partial s_{i1}} = \lambda_1 p_{i1}^H + \mu_1 \left[p_{i1}^H - p_{i1}^L\right] - \sum_{j=1}^{n} \nu_{ij} + \eta_i = 0.$$

Since the agent is assumed to earn a rent, it holds that $\lambda_1 = 0$. The incentive constraint (10) will be binding, therefore $\mu_1 > 0$. Thus, if $p_{i1}^H > p_{i1}^L$ it holds that $\eta_i > 0$, i.e. the liability constraint is binding and $s_{i1} = s^{min}$.

To prove claim 2, consider the first-order condition with respect to the second-period compensation s_{ij2},

$$\frac{\partial \mathcal{L}}{\partial s_{i1}} = \lambda_1 p_{i1}^H p_{j2}^H + \lambda_{i2} p_{j2}^H + \mu_1 p_{i1}^H \left[p_{j2}^H - p_{j2}^L\right] - \nu_{ij} + \eta_{ij} = 0. \quad (22)$$

Again, $\lambda_1 = 0$ holds by our assumption that the agent earns a rent. Moreover, the liability constraint $s_{ij2} \geq s^{min}$ will not be strictly binding for the considered case that $s_{i1} > s^{min}$ because if the principal would like to decrease s_{ij2}, he could likewise decrease s_{i1}, which has the same effect. Therefore, $\eta_{ij} = 0$ and (22) can be written as

$$\lambda_{i2} p_{j2}^H + \mu_1 p_{i1}^H \left[p_{j2}^H - p_{j2}^L\right] - \nu_{ij} = 0. \quad (23)$$

Now assume that the budget constraint (12) is not binding and $\nu_{ij} = 0$. Condition

(23) becomes

$$-\lambda_{i2} = \mu_1 p_{i1}^H \left[1 - \frac{p_{j2}^L}{p_{j2}^H}\right],$$

which can only be fulfilled for one single realizations of Y_2 because of the strict monotone likelihood ratio property. Since $-\lambda_{i2} < \mu_1 p_{i1}^H \left[1 - p_{j2}^L / p_{j2}^H\right]$ will hold for all other realizations by $\nu_{ij} > 0$, this single outcome has to be the one with the highest likelihood ratio, $j = 1$. Thus, the bonus pool constraint will be binding for all but the worst performance. Given this binary incentive scheme, the necessary wage spread can be derived from the second-period incentive constraint which becomes

$$s_{i12} + (1 - p_{12}^H)\left[\overline{s}_{i2} - s_{i12}\right] - c_t^H \geq s_{i12} + (1 - p_{12}^L)\left[\overline{s}_{i2} - s_{i12}\right]\overline{s}_{i2}$$

or

$$\left[p_{12}^L - p_{12}^H\right]\left[\overline{s}_{i2} - s_{i12}\right] \geq c_t^H$$

from which the cost-minimizing bonus

$$\overline{s}_{i2} - s_{i12} = \frac{c_t^H}{p_{12}^L - p_{12}^H}$$

follows. □

Proof of Proposition 4:

The binary structure of the contract can be derived by the same line of arguments as in the proof to Proposition 1, just substituting compensation terms s by monetary utilities $v(s)$. Given the binary structure, the incentive constraint becomes

$$P_{t-}^H v(s^{min}) + P_{t+}^H v(\overline{s}_t) - c_t^H \geq P_{t-}^L v(s^{min}) + P_{t+}^L v(\overline{s}_t)$$

or

$$\left[P_{t+}^{H}-P_{t+}^{L}\right]\left[v(\overline{s}_{t})-v(s^{min})\right]\geq c_{t}^{H}$$

from which the cost-minimizing bonus pool of size

$$\overline{s}_{t}=v^{-1}\left(v(s^{min})+\frac{c_{t}^{H}}{P_{t+}^{H}-P_{t+}^{L}}\right)$$

can be derived. □

To prove Proposition 5, it is helpful to first consider the following lemmas:

Lemma 2 *If the likelihood ratio p_i^L/p_i^H of probability functions p_i^k is strictly decreasing in i, this likelihood ratio for $i<n$ is strictly larger than the likelihood ratio P_i^L/P_i^H of the corresponding survival functions $P_i^k=\sum_{j=i}^{n}p_j^k$, i.e.,*

$$p_i^L/p_i^H > P_i^L/P_i^H \quad \forall\, i<n.$$

Proof Since $p_n^k = P_n^k$, it suffices to show that from $p_{i+1}^L/p_{i+1}^H \geq P_{i+1}^L/P_{i+1}^H$ and strict MLRP it follows that $p_i^L/p_i^H > P_i^L/P_i^H$. Strict MLRP and $p_{i+1}^L/p_{i+1}^H \geq P_{i+1}^L/P_{i+1}^H$ imply that

$$\frac{p_i^L}{p_i^H} > \frac{P_{i+1}^L}{P_{i+1}^H} \quad \text{or} \quad \frac{P_{i+1}^H}{p_i^H} > \frac{P_{i+1}^L}{p_i^L}.$$

Adding 1 on both sides yields

$$\frac{p_i^H+P_{i+1}^H}{p_i^H} > \frac{p_i^L+P_{i+1}^L}{p_i^L}$$

which due to the fact that $p_i^k + P_{i+1}^k = P_i^k$ gives

$$\frac{P_i^H}{p_i^H} > \frac{P_i^L}{p_i^L} \quad \Leftrightarrow \quad \frac{p_i^L}{p_i^H} > \frac{P_i^L}{P_i^H}.$$

$\square$

Lemma 3 *If the likelihood ratio p_i^L/p_i^H of probability functions p_i^k is strictly decreasing in i, the same holds for the likelihood ratio P_i^L/P_i^H of survival functions $P_i^k = \sum_{j=1}^{i} p_j^k$.*

Proof From Lemma 2 we know that

$$\frac{p_i^L}{p_i^H} > \frac{P_i^L}{P_i^H} \quad \forall\, i < n$$

or

$$\frac{p_i^L}{p_i^H} \geq \frac{P_i^L}{P_i^H} \quad \forall\, i.$$

By MLRP it follows that

$$\frac{p_i^L}{p_i^H} > \frac{P_{i+1}^L}{P_{i+1}^H} \Leftrightarrow \frac{p_i^L}{P_{i+1}^L} > \frac{p_i^H}{P_{i+1}^H} \quad \forall\, i < n.$$

Adding 1 on both sides gives

$$\frac{p_i^L + P_{i+1}^L}{P_{i+1}^L} > \frac{p_i^H + P_{i+1}^H}{P_{i+1}^H} \quad \forall\, i < n.$$

Using the fact that $p_i^k + P_{i+1}^k = P_i^k$ we get

$$\frac{P_i^L}{P_{i+1}^L} > \frac{P_i^H}{P_{i+1}^H} \Leftrightarrow \frac{P_i^L}{Pi^H} > \frac{P_{i+1}^L}{P_{i+1}^H} \quad \forall\, i < n.$$

□

Proof of Proposition 5:

The proof is by construction. Suppose that the short-term contract, (21), is used in both periods. We show that the principal can do better.

To that purpose, consider the following variation of the second-period contract

$$\Delta s = \begin{cases} \Delta_1 & \text{for} \quad j = \hat{j} = \min I_2^+ \\ \Delta_2 & \text{for} \quad j \in I_2^+ \backslash \hat{j}. \end{cases} \tag{24}$$

for the case that the agent's performance in the first-period was bad news: His compensation for the lowest level $\hat{j} = \min I_2^+$ of performance obeying good news is decreased by Δ_1, while the compensation for all other good news performance levels $j > \hat{j}$ are increased by Δ_2, where Δ_1 and Δ_2 are chosen to fulfil

$$v(s_{\hat{j}2}) = v(\overline{s}_2) - \delta \tag{25}$$

and

$$v(s_{j2}) = v(\overline{s}_2) + \delta \frac{p_{\hat{j}2}^H - p_{\hat{j}2}^L}{P_{\hat{j}+1,2}^H - P_{\hat{j}+1,2}^L}, \tag{26}$$

where $P_{\hat{j}+1,2}^k = \sum_{j=\hat{j}+1}^n p_{j2}^k$ is the probability that the agent's second-period performance is at least $y_{\hat{j}+1}$. Since we consider the case in which no bonus was paid in the first period, such a variation always exists for levels of δ small enough.

Equations (25) and (26) guarantee that the agent's second-period incentive constraint is still met. The agent's second-period expected utility, however, differs by

$$-\delta p^H_{\hat{j}2} + \delta \frac{p^H_{\hat{j}2} - p^L_{\hat{j}2}}{P^H_{\hat{j}+1,2} - P^L_{\hat{j}+1,2}} P^H_{\hat{j}+1,2}.$$

This amount is negative if

$$\frac{p^H_{\hat{j}2}}{p^H_{\hat{j}2} - p^L_{\hat{j}2}} > \frac{P^H_{\hat{j}+1,2}}{P^H_{\hat{j}+1,2} - P^L_{\hat{j}+1,2}} \Leftrightarrow \frac{p^H_{\hat{j}2} - p^L_{\hat{j}2}}{p^H_{\hat{j}2}} < \frac{P^H_{\hat{j}+1,2} - P^L_{\hat{j}+1,2}}{P^H_{\hat{j}+1,2}} \Leftrightarrow \frac{p^L_{\hat{j}2}}{p^H_{\hat{j}2}} > \frac{P^L_{\hat{j}+1,2}}{P^H_{\hat{j}+1,2}}$$

which is always the case under MLRP (see Lemma 3).

All other things equal, the agent therefore incurs a utility loss if he fails in the first period, compared to the situation with two short-term contracts. Thus, the principal may decrease the first-period bonus $\bar{s}_1$ without violating the first-period incentive constraint. By this means, the total bonus pool size and thus the principal's compensation cost is decreased. □

References

[1] Baker, G., R. Gibbons, and K. Murphy (1994) Subjective Performance Measures in Optimal Incentive Contracts. Quarterly Journal of Economics 109, 1125-1156.

[2] Bull, C. (1987) The Existence of Self-enforcing Implicit Contracts. Quarterly Journal of Economics 102, 147-159.

[3] Baiman, S. and M. Rajan (1995). The Informational Advantages of Discretionary Bonus Schemes. The Accounting Review 70, 557–579.

[4] Demougin, D. and C. Fluet. (1998). Mechanism Sufficient Statistic in the Risk-Neutral Agency Problem. Journal of Institutional and Theoretical Economics 154, 622–639.

[5] Ederhof, M. (2010). Discretion in Bonus Plans. The Accounting Review 85 (6), 1921–1929.

[6] Ederhof, M., M.V. Rajan and S. Reichelstein (2010). Discretion in Managerial Bonus Pools. Working paper.

[7] Gibbs, M., K.A. Merchant, W.A. Van der Stede and M.E. Vargus (2004). Determinanats and Effects of Subjectivity in Incentives. The Accountung Review 79 (2), 409–436.

[8] Innes, R.D. (1990). Limited liability and incentive contracting with ex-ante action choices. Journal of Economic Theory 52, 45–67.

[9] Kim, S.K. (1997). Limited liability and bonus contracts. Journal of Economics and Management Strategy 6, 899–913.

[10] MacLeod, B. (2003). Optimal Contracting with Subjective Evaluation. American Economic Review 93, 216–240.

[11] Murphy, K. and P. Oyer (2003). Discretion in Executive Incentive Contracts. Working paper.

[12] Rajan, M.V. and S. Reichelstein (2006). Subjective Performance Indicators and Discretionary Bonus Pools. Journal of Accounting Research 44, 585–618.

[13] Rajan, M.V. and S. Reichelstein (2009). Objective versus Subjective Indicators of Managerial Performance. The Accounting Review 84, 209–237.