Multi-Stage Resource Allocation under Asymmetric Information

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Abstract

Investments frequently involve multiple rounds of funding; however, traditional models of the capital allocation process have considered either single-period/single-project models or multi-period models in which the investments each period are independent. In contrast, we examine a single-period but multi-stage model in which, at Stage 1, a manager proposes an initial project (or experiment) which the firm either accepts or rejects. If the project is initially accepted, its continuation value becomes known to the firm and manager at the end of Stage 1, at which point the firm determines whether or not to continue the project in Stage 2. We show that in this setting, the firm optimally commits to a capital allocation scheme that exhibits underinvestment at Stage 1, but exhibits instances of both over and underinvestment at Stage 2. That is, at Stage 1 the firm foregoes positive NPV projects, while at Stage 2 it implements negative NPV projects in some instances and foregoes implementing positive NPV projects in others. We provide further characterization of the optimal capital budgeting process and comparative statics.

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I. Introduction

One of the most important determinants of a firm’s success is the effectiveness of its capital allocation decisions. By channeling funds across different initiatives, capital allocation decisions translate a firm's strategic plan into strategic investments. In addition, few decisions have as profound an impact on the promotion, compensation, and status of managers vying for capital. Not surprisingly, Wulf [2002, p.2] empirically finds that “…there is rent seeking behavior by division managers…” and that these managers “…have an incentive to engage in influence activities and distort subjective information about investment opportunities…” This agency problem was an early, and remains a continuing, subject of incentive contracting models (e.g., Harris et al. [1982], Bernardo et al. [2009]).

Prior models of the capital allocation process have traditionally considered either single-period models in which the manager proposes — and the principal accepts or rejects — a single capital project, or multi-period models in which the manager proposes independent projects each period. In contrast, we examine a single-period, multi-stage model in which the manager proposes an initial project (or experiment) which the principal either accepts or rejects. If the project is initially accepted, its continuation value becomes known to the principal and manager at the end of Stage 1, at which point the principal determines whether to continue or abandon the project in Stage 2.

Our setting is descriptive of the staged capital allocation process found in private equity, venture capital, and intra-firm R&D investments (see Gompers [1995]). These types of capital allocation decisions are multi-stage for several reasons. For example, R&D investments may contain multiple investment stages because additional technological information (regarding cost, reliability and scalability) or market information (regarding availability of suppliers or demand) can only be acquired after constructing a prototype plant or product. Another applicable setting is one where regulatory obligations must be satisfied sequentially; as is the case with newly developed drugs undergoing FDA approval. In our two-stage capital allocation setting with a privately informed manager, we show that the firm optimally commits to a capital allocation scheme that exhibits underinvestment at Stage 1 (the project-initiation or experiment stage), but exhibits instances of both over and underinvestment at Stage 2 (the full project implementation stage). That is, at Stage 1 the firm foregoes positive NPV projects, while at Stage 2 it implements negative NPV projects in some instances and foregoes positive NPV projects in others. The propensity to over or underinvest depends on the manager’s report about the Stage 1 investment opportunity and the outcome of his Stage 1 investment. Our findings are consistent with the empirical literature including

Poterba and Summers [1995] and Driver and Temple [2009] who find that while on average firms employ capital budgeting hurdle rates in excess of their cost of capital (resulting in underinvestment), a non-trivial percentage use hurdle rates below their cost of capital (resulting in overinvestment). Our results are also the first to provide a pure adverse selection explanation for under- and overinvestments within the same firm as documented in the empirical internal markets literature; e.g., Lamont [1997].

The paper is organized as follows. In Section II we review the relevant literature. In Section III we present the model and our major results. We illustrate our findings in Section IV with an example before concluding in Section V.

II. Literature Review

The present paper builds on the single-period, adverse selection capital budgeting model of Antle, and Eppen [1985]. In their paper, project revenues are commonly known, but the investment required to generate the revenues is privately known by the manager. The manager's sole role is to report the required investment to the principal. Because the manager is assumed to consume any allocated funds above and beyond those required for the project, his incentive is always to overstate the project's true cost. To mitigate the manager's incentive for overstating the cost, the optimal capital allocation rule couples a simple funding rule with an investment hurdle: projects with reported costs in excess of the hurdle are rejected, whereas those below the hurdle are funded and receive the same fixed budget.[1] Further, the optimal hurdle rate exceeds the firm's cost of capital, causing the firm to reject some positive NPV projects in order to economize on the manager's informational rents.[2]

Subsequent work has proposed several variations to the Antle-Eppen model. For example, Antle, and Fellingham [1990], Fellingham, and Young [1990] and Arya et al. [1994] extend the model to a repeated game, where the manager privately observes and reports on a new investment opportunity in each period. By committing to history-dependent capital allocation rules, the principal both reduces the manager's informational rents and mitigates the underinvestment problem. Our setting is fundamentally dif-

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[1] In contrast, Bockem, and Schiller [2009] is an exception in that when the manager must also be motivated to acquire his private information, the range of funded projects will vary in his report. A similar extension is considered in Kim [2006].

[2] The optimality of hurdle rates has also been shown in capital budgeting problems without incentive conflicts; e.g., Baldwin [1982] and Prastacos [1983].

ferent from this extension. Specifically, this literature assumes that the investment opportunity in each period is self-contained – the expected cash flow from funding the period *n*-1 investment opportunity is the same whether the investment opportunity in period *n* is funded, and vice-versa. In contrast, we consider projects where the Stage 1 investment informs the firm as to the investment opportunity available in Stage 2. Further, the investment in Stage 2 is only possible if the firm previously invested in Stage 1; i.e., the former can be viewed as an expansion or continuation of the latter.

Another variation to the capital budgeting literature involves the incorporation of real options. For example, in Antle et al. [2006], the firm can invest in a project discovered and reported on by the manager in the present period, or postpone the investment to a later period after the manager reports a new investment opportunity. The option to delay investments allows the principal to more efficiently motivate the manager to reveal his private information. In our model, rather than giving the principal the option to delay investing, we endow her with the opportunity to continue or abandon the project upon receiving interim information at the end of Stage 1.

Further, although our model is similar to those discussed above in that we have a pure adverse selection problem, our results are quite different. In particular, most of the pure adverse selection models in the capital budgeting literature find that the optimal capital allocation process has two consistent characteristics: it is optimal to ration capital so that the firm underinvests relative to First-Best; and the initial level of funding is independent of the manager’s report. In contrast we find that when the projects have an abandonment option, the principal continues to underinvest at Stage 1, though the level of funding now varies with the manager’s report. Further, we find that the optimal Stage 2 capital allocation decision can result in instances of both overinvestment and underinvestment, depending on the manager's initial report. The latter result has been found to hold empirically, as individual firms have been shown to both under- *and* overinvest across different projects. In fact, the distortions that we find are closer to, although not the same as, those derived from capital allocation models where the manager is subject to both adverse selection and moral hazard.

The paper whose findings are closest to ours in the adverse selection/moral hazard literature is Bernardo, Cai, and Luo [2009]. They examine a single-stage setting where the principal must motivate the manager to: (i) expend “entrepreneurial” effort to privately discover investment opportunities; (ii) report that information to the principal; and (iii) expend “managerial” effort to manage the project should the principal choose to fund it. The optimal capital allocation rule exhibits both under and overinvestment. In a separate study, Bernardo et al. [2006] consider a multi-division firm where the managers are privately informed and engage exclusively in managerial effort. In this simpler setting, the authors find that in expectation, the principal favors investment in the weaker division at the cost of investment in the stronger division. However, the optimal hurdle rates for both the weak and strong divisions are above the cost of capital. Similar to our model, they predict varying hurdle rates depending on the managers' report, although unlike our model, they do not find instances of overinvestment. Dutta, and Fan [2009] study a model similar to Bernardo, Cai, and Luo [2009] but where the manager only expends entrepreneurial effort. In their single-stage model, they find that either under- or overinvestment may occur. Taken together, these three papers suggest that for capital budgeting models with adverse selection and moral hazard, entrepreneurial effort is necessary for overinvestment to occur in equilibrium. In contrast, our multi-stage model identifies optimal overinvestment without any embedded moral hazard problems.

Like our paper, Pfeiffer, and Schneider [2007] consider a multi-stage capital allocation problem with abandonment options. Our models are similar in that the manager is unable to consume his entire informational rent unless the project is fully implemented. However, Pfeiffer and Schneider impose a moral-hazard problem in Stage 1 where the manager engages in hidden managerial effort. Further, if the principal abandons the project at Stage 2, she can perfectly (and verifiably) document the manager's prior managerial effort choice. Thus, in Pfeiffer, and Schneider [2007], abandoning the investment yields a positive externality, in that it allows the principal to attain the First-Best outcome with respect to the moral hazard problem. In contrast, in our model abandoning the project generates a *negative* externality, as it imposes a deadweight loss to the principal who is unable to recover all funds invested at Stage 1 in excess of the manager's cost.[3]

III. Model

We study a capital budgeting model with sequential information arrival and funding. In particular, we consider a model with two stages: in Stage 1, the manager is privately informed as to the cost of initiating a project; if Stage 1 is funded, interim project information becomes available at Stage 2, when the investment required for continuation becomes publicly known. We label Stage 1 the *research* or *experiment stage* and Stage 2 the *implementation stage*, as the project is assumed to end without any payoff

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[3] In a related paper, Johnson et al. [2010] extend the analysis in Pfeiffer, and Schneider [2007] to investments which can be used my multiple divisions within the firm. See also, Vaysman [2006], for a model where abandoning the project causes the principal to forgo positive profits.

in Stage 2 unless the principal makes the second investment. Consistent with the two stages we will refer to an investment in Stage 1 as "funding" a project and an investment at Stage 2 as "implementing" a project.

At time *t*=1 (see Figure 1) the risk-neutral firm (the principal) hires a risk-neutral manager to manage both stages of the project. Once the manager is hired, both he and the principal (she) know that a project implemented in *t*=4 will generate a gross cash flow of R at *t*=5, although at *t*=1, they are equally uncertain as to the additional investment required to implement the project. At time *t*=0, prior to being hired, the manager is privately informed as to the level of funding required to carry out the research stage (Stage 1). We assume that the research stage requires a minimum investment of $c_i \in C = \{c_1, ..., c_n\}$, where $c_i < c_{i+1}\ \forall i$. In Stage 1, any investment greater than or equal to c_i generates a perfect signal about the additional investment cost, $m_j \in M = \{m_1, ..., m_k\}$, required to fully implement the project at *t*=4. Without loss of generality, any Stage 1 investment less than the observed c_i will cause the research stage to fail with certainty; i.e., it will not generate a signal. The difference $G_j = R - m_j$ thus denotes the continuation value of the project beyond *t*=3. The probability densities of c and m are common knowledge, independent and represented by $f(c_i)$ and $f(m_j) \equiv \rho_j$ respectively.[4] In order to facilitate the analysis, we assume that the discrete cost signals, c_i, are evenly spaced, with $c_i - c_{i-1} = \delta$ for $i > 1$ and $c_1 = \delta$. Second, we assume that Stage 1 costs are uniformly distributed: $f(c_i) = \frac{1}{n}$, though the discrete density of the Stage 2 costs, ρ_j, and its support, M, are unrestricted.

As an example, consider an automotive firm interested in developing a new battery technology with a known market potential (R) but unknown cost of production (m_j). The firm hires a scientist to set up a lab in order to develop the technology in-house, although only the scientist knows the cost of developing the technology (c_i). While the scientist cannot conduct the necessary research without sufficient funding, any funding in excess of c_i does not benefit his research. The scientist invests any funding in excess of c_i in research equipment which will allow him to conduct tangential experiments which are of no value to the firm but are of interest to the scientist, thereby creating perquisites for the scientist. We assume that the scientist will only have time to conduct this tangential research and consume the per-

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[4] See the Appendix for a listing of notation.

quisites during Stage 2, and therefore, only if Stage 2 implementation investment is made.[5] Given sufficient Stage 1 funding (c_i), the scientist's research results in a prototype which informs the firm as to the cost of mass producing the battery (m_j), which could potentially exceed the payoff to commercializing the project (R); i.e., although sufficiently funded research is always successful in uncovering information, the results need not be favorable.

The budgeting and compensation schedules are as follows. In period *t*=1, the principal proposes a contract which specifies all compensation and investments until the end of the final period (*t*=5). At time *t*=2, the manager uses his private information to submit a report, $\hat{c}_i$, to the principal about the required minimum research investment and the latter funds the manager according to the contract set forth in period *t*=1: with probability $z(\hat{c}_i) \in [0,1]$ the project is funded and with probability $1 - z(\hat{c}_i)$ it is not. If the project is not funded, the game ends. On the other hand, if the project is funded, the manager receives a budget of $b(\hat{c}_i)$ to invest at the outset of *t*=3. The manager invests his entire budget in research related assets. However, only c_i is dedicated to the research project and the remaining $b(\hat{c}_i) - c_i$ is invested in other assets, the use of which can provide the manager with personal utility. Unlike the prior literature, we assume that the manager cannot withdraw $b(\hat{c}_i) - c_i$ in the form of cash for personal consumption.[6] We assume that the manager will only have time to consume the excess assets if the project is implemented in Stage 2. At the end of *t*=3, both the principal and manager observe the contractible research outcome, m_j, which denotes the principal's required investment to implement the project in Stage 2. At time *t*=4, the principal continues the project in accordance with the contract agreed to at *t*=1: she implements the project with probability $p(\hat{c}_i, m_j) \in [0,1]$, and abandons it with probability $1 - p(\hat{c}_i, m_j)$.[7] At *t*=4, the earlier investments are sunk, therefore if the project is abandoned at *t*=4, then neither the principal nor the manager benefit from the *t*=2 investment or surplus. On the other

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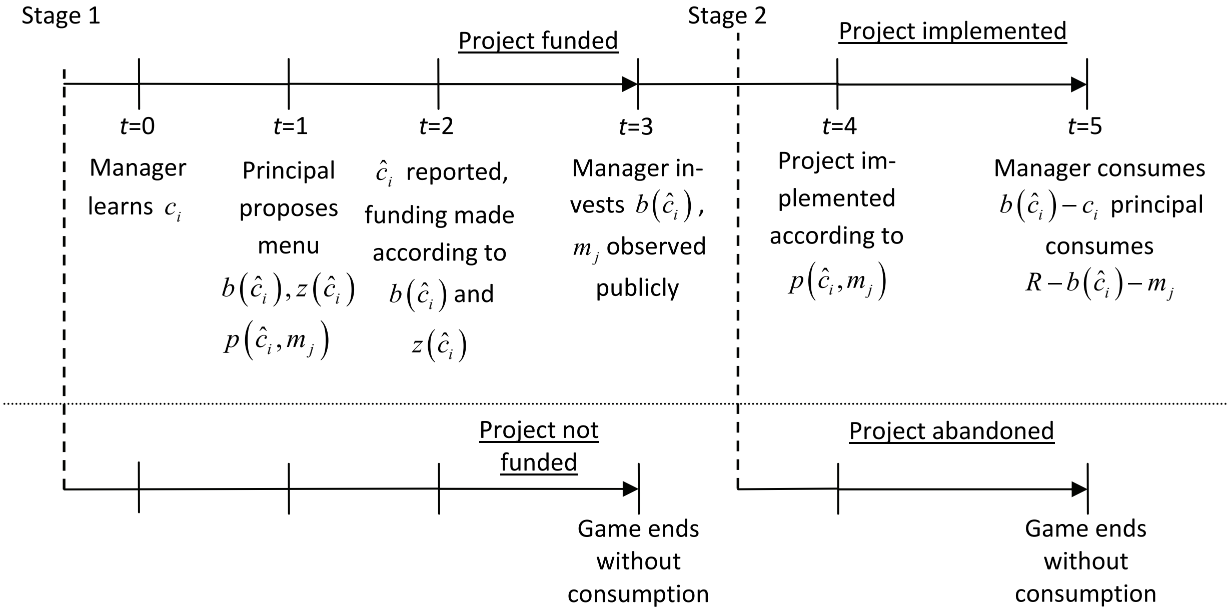
[5] Bolton, and Dewatripont [1994] make a similar assumption that the manager receives private benefits from a project only if it is not canceled by the debt holders.

[6] For example, the presence of a purchasing office, organizational purchasing rules or the threat of audit are sufficient to prevent the manager from directly consuming unspent funds.

[7] Milestone contingent financing (staging) is frequently observed in practice, both in the venture capital industry, and inside firms who finance internal R&D (see Gompers [1995]).

hand, if the project is implemented at *t*=4, then in *t*=5, the principal consumes the residual surplus, $R-b(\hat{c}_i)-m_j$, and the manager consumes the excess funding procured in Stage 1: $b(\hat{c}_i)-c_i$. The sequence of events is displayed in Figure 1.

Figure 1: Timeline



Our setting can be seen as a reduced form of a setting in which: (i) the agent can consume some portion (but not all) of the excess funds allocated in Stage 1, but can only consume the remaining excess if the project is later implemented in Stage 2, and (ii) the principal can recoup some portion (but not all) of the initially invested funds if she decides to abandon the project at Stage 2. All qualitative results continue to hold as long as three critical assumptions are met: (1) the manager does not consume all the slack in Stage 1, (2) some value to the principal is lost upon salvaging the assets (the principal cannot recoup all funds invested if the project is abandoned), and (3) the principal and manager cannot contract on the sale of abandoned assets. Assumptions (1) and (2) guarantee that the principal incurs a deadweight loss if the project is abandoned, whereas (3) assures that the principal cannot fully resolve the Stage 1 adverse selection problem should she later abandon the project.[8]

In order to study the efficiency of the model described above, we first establish two benchmark models. The *First-Best model* is identical to the above, albeit without adverse selection; that is, we assume that

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[8] See Pfeiffer, and Schneider [2007] for a setting in which project abandonment allows the principal to verify the manager's hidden efforts.

the manager's research cost, c_i, is common knowledge. The second benchmark model is identical to the model above but we assume that the principal cannot commit to a Stage 2 implementation rule. Instead, the principal chooses a sequentially rational Stage 2 implementation rule, implying that she will implement all projects whose continuation cost, m_j, is less than the revenue, R, independent of the Stage 1 cost. We refer to this second benchmark as the *No Commitment model*. Notice that in the No Commitment model, the principal cannot commit to a (non-sequentially rational) Stage 2 implementation rule which would encourage the manager to truthfully report his Stage 1 private information. This second benchmark allows us to highlight the principal's incentive to distort Stage 2 implementations to mitigate the information asymmetry faced in Stage 1.

Returning to our model, upon reporting $\hat{c}_i$, the manager's expected utility is given by :

$$U_A = \sum_{j=1}^{k} p\left(\hat{c}_i, m_j\right) \rho_j \, z\left(\hat{c}_i\right)\left(b\left(\hat{c}_i\right) - c_i\right).$$

Because the manager can only consume slack if the project is implemented, his expected utility depends on both the principal's Stage 1 and Stage 2 decisions. We use the Revelation Principle to restrict our analysis to contracts which induce the manager to truthfully report his private cost information without loss of generality, that is, we only study contracts which optimally induce $\hat{c}_i = c_i$. To simplify what follows, let $b_i \equiv b\left(c_i\right)$, $p_{i,j} \equiv p\left(c_i, m_j\right)$, and $z_i \equiv z\left(c_i\right)$. Thus, at time *t*=1, the principal maximizes:

$$U_P = \sum_{i=1}^{N} z_i \left[-b_i + \sum_{j=1}^{k} \rho_j \left(R - m_j\right) p_{i,j} \right] f\left(c_i\right)$$

subject to:

$$z_i \sum_{j=1}^{k} \rho_j p_{i,j} \left(b_i - c_i\right) \geq z_s \sum_{j=1}^{k} \rho_j p_{s,j} \left(b_s - c_i\right) \quad i = 1, \ldots, n \tag{IC}$$

$$z_i \sum_{j=1}^{k} \rho_j p_{i,j} \left(b_i - c_i\right) \geq 0 \quad i = 1, \ldots, n. \tag{IR}$$

The incentive compatibility constraint (IC) ensures that the manager truthfully reports his cost signal, c_i, whereas the individual rationality constraint (IR) guarantees that the manager receives his outside reservation wage, which we have normalized to zero without loss of generality.[9] Note that, conditional on a Stage 1 cost report $\hat{c}_i$, the manager is only interested in the interim information, m_j, to the extent that it affects the probability of his project being implemented. Thus, when applicable, we let $p_i \equiv \sum_{j=1}^{k} \rho_j p_{i,j}$ denote the probability of project implementation at time *t*=4 conditional on the cost report, $\hat{c}_i$. The principal has three choice variables with which to control the manager's incentives: the set of cost reports for which the Stage 1 research will be funded (the set of c_i for which $z_i > 0$); the budget for those funded experiments (b_i); and the Stage 2 implementation rule ($p_{i,j}$). Proposition 1 below characterizes the principal's optimal Stage 1 funding rule.

Proposition 1: The optimal Stage 1 funding rule:

(i) Defines a threshold cost report, c_h,[10] such that projects are always funded ($z_i = 1$) when the reported cost falls below the threshold ($\hat{c}_i \leq c_h$) and always rejected ($z_i = 0$) when the reported cost exceeds the threshold ($\hat{c}_i > c_h$).

(ii) Provides the manager with a budget: $b_i = c_i + \delta \sum_{q=i+1}^{h} \frac{p_q}{p_i}$, which is weakly increasing in the manager's reported cost, $\hat{c}_i$.[11]

Note that we obtain "weak" results in Prop. 1 (ii) and throughout the paper due to the discreteness of our densities. All results will be strict if we assume that the Stage 1 and Stage 2 costs are each distributed with sufficiently small distances between the adjacent costs (see the example in Section IV).

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[9] Without loss of generality, we ignore the possibility that the principal also compensates the manager when: (a) the principal does not invest in the research stage and (b) the principal does invest in the research stage, but chooses not implement the project. The proof of this assertion is relatively long but follows from the assumed risk-neutrality of the manager and lack of any additional agency problems.

[10] In what follows, we denote the threshold Stage 1 cost derived from our setting c_h, as distinct from the optimal threshold cost c_h^{FB}, which emerges from the First-Best model, and the optimal threshold cost c_h^{NC}, which emerges from the No Commitment model.

[11] All proofs are in the Appendix.

Proposition 1 (i) re-establishes the familiar hurdle contract found in the earlier single-stage investment models. Proposition 1 (ii) shows that in our setting, budgets are *increasing* in the reported cost, $\hat{c}_i$, thereby giving the manager an incentive to overstate his private cost observation. This result depends upon the implementation probabilities chosen by the principal being decreasing in the manager's initial cost report, which we show to be true in Proposition 2.

Recall that in the No Commitment model, sequential rationality requires that if a project is funded, it is implemented with the same probability as all other funded projects; i.e., $p_i = \sum_{j=1}^{r} \rho_j$, where m_r is the greatest continuation cost less than or equal to R. If one were to use these implementation probabilities in our model, then the optimal budgets found in part (ii) of Proposition 1 become identical for all Stage 1 reports, which agrees with the optimal simple funding rule in prior single-stage capital budgeting models (e.g., Antle, and Eppen [1985]).

As noted above, with commitment, the optimal Stage 1 budget rule provides an incentive for the manager to over-report his Stage 1 cost observation. In contrast to the prior literature, Proposition 2 shows that the principal counteracts this incentive by making the probability of Stage 2 implementation (and therefore the probability that the manager can consume his slack) *decrease* in the manager's cost report.

Proposition 2: The optimal Stage 2 implementation rule involves:

(i) A threshold value, $\overline{m}_i \equiv \overline{m}(c_i)$ for each Stage 1 cost announcement, $\hat{c}_i$, such that all projects with sufficiently small continuation costs ($m < \overline{m}_i$) are implemented with certainty ($p_{i,j} = 1$), those with intermediate continuation costs, $m = \overline{m}_i$, are implemented with non-zero probability ($p_{i,j} \in (0,1]$), and those with larger continuation costs, $m > \overline{m}_i$, are never implemented ($p_{i,j} = 0$).

(ii) The probability of implementation, $p_{i,j}$, is weakly decreasing in both the project continuation cost, m_j, and the manager's reported cost observation, $\hat{c}_i$.

Proposition 2 (i) shows that in Stage 2, the principal employs a threshold strategy based on the continuation cost, m_j, similar to the rule applied in Stage 1 with respect to the manager's reported cost. While the actual continuation cost is immaterial to the manager, he is *indirectly* interested in it, because it affects the project continuation probability, $p_{i,j}$, and hence the likelihood that in Stage 2 he can consume the excess funding made in Stage 1. The principal's profits, however, are *directly* a function of the continuation costs. Thus, greater continuation costs are paired with lower probabilities of implementation. The last part of (ii) reflects the other side of the principal's objective; balancing managerial rents. Because greater reported costs receive larger budgets, the principal must penalize such reports with reduced implementation probabilities; otherwise all managers would claim to have observed the largest private cost realization. Combined with Proposition 1 (ii), this latter result implies that our budgets are weakly less than those obtained in the No Commitment model.[12]

Propositions 1 and 2 together imply that a manager reporting a higher private cost realization receives a larger budget, but his project is implemented with a lower probability. The pairing ensures that (IC) is satisfied and allocates greater expected rents to managers reporting less costly observations. The manager's reporting problem is thus similar to the trade-off faced by a bidder in both first- and second-price auctions: low bids guarantee large payoffs if the bidder wins the auction, though the probability of winning is increasing in the bid. Unlike a traditional auction, our principal finds it optimal to allocate non-binary probabilities to implementing projects. Indeed we find that balancing her two competing objectives, the principal may optimally randomize Stage 2 implementation with a probability between 0 and 1 (this can be seen in Proposition 2 (i) where $p_{i,j} \in (0,1]$ for $m = \overline{m}_j$). However, note that this randomization can only occur when the precise Stage 2 threshold continuation costs is realized. Having solved for the optimal Stage 2 implementation rule, we next characterize the resulting distortions.

Proposition 3: Relative to the No Commitment model, the principal's optimal contract with commitment will:

(i) Weakly overinvest in Stage 2 when the manager reports the lowest Stage 1 costs.

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[12] Antle and Fellingham [1990] derive a somewhat similar Stage 1 funding result, but only for specific parameters. Adding a second round of investing in Antle-Fellingham produces an optimal Stage 1 funding rule which divides the set of projects into three categories: those that aren't funded, those that are and receive the same budget, and one which is funded and receives a smaller budget than the others. Our two-stage investment process allows us to more finely tune the Stage 1 funding rule.

(ii) Weakly underinvest in Stage 2 when the manager reports large Stage 1 costs.

(iii) Weakly underinvest in Stage 2 on average; i.e., $\frac{1}{h}\sum_{i=1}^{h}\sum_{j=1}^{k} p_{i,j}\rho_j m_j \le R$.

Sequential rationality at Stage 2 requires that the principal implement all projects with a non-negative continuation value, $\overline{m}_i \le R \; \forall i \le h$, regardless of the Stage 1 reported cost or budget. However, Proposition 3 shows that if overinvestment is possible at the implementation stage (i.e., $R < m_k$) the principal may find it optimal to commit to *overinvest* at Stage 2 ($\overline{m}_l > R$) for sufficiently small reported Stage 1 costs.[13] For higher reported Stage 1 costs, the principal will commit to underinvest at Stage 2, as $R > \overline{m}_h$, and the thresholds $\overline{m}_i$ are decreasing in i as per Proposition 2 (ii). However, on average, the principal underinvests in Stage 2 implementations.

This implies that if one only observed project implementation decisions in Stage 2, one could observe a seemingly irrational strategy where capital is allocated to projects with a continuation value of $R - m_j$ and denied to those with continuation value of $R - m_{j-i}$, in spite of the former being less profitable than the latter ($m_{j-i} < m_j$). Although our model consists of a single division, the result extends to a multi-division setting in which the divisions are stochastically independent. In that case, the results would be consistent with empirical findings of biased capital allocation whereby "weaker" divisions sometimes receive more funding than stronger divisions (e.g., Lamont [1997]).[14]

We note that Proposition 3 (i) stands in contrast to the traditional pure adverse-selection literature whereby only high cost managers are optimally subject to production distortion. In order to understand why our result differs, recall that in the majority of the prior adverse selection literature, the manager always consumes his informational rents (as in a pure exchange model). However in our setting, the manager only consumes the extracted rent if the principal eventually implements the project in Stage 2. If the principal does not implement the project (which happens with probability $1 - p_i$), then any resi-

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[13] See Section IV for an example with optimal overinvestment in Stage 2.

[14] Proposition 3 thus agrees with Bernardo et al., 2006: profitable projects are forfeited to seemingly subsidize less profitable ones. Unlike Bernardo et al., 2006, the endogenous over and underinvestment do not follow from an underlying moral hazard problem, but from the principal's desire to mitigate the manager's informational rents.

dual managerial slack, $b_i - c_i$, is transformed into a deadweight loss, and no one benefits from the overage. To emphasize the role of the deadweight loss in our result, note that if the manager can consume all available slack in Stage 1 or if the principal can sell any remaining assets in Stage 2 at no loss, then overinvestment in Stage 2 is no longer optimal. Thus in addition to the traditional tradeoff between production efficiencies and informational rents, our principal is also concerned with the deadweight loss associated with the Stage 2 abandonment of funded projects.[15] To understand the efficacy of the Stage 2 distortions in deterring the manager from over-reporting his cost, we examine the principal's distortions of the optimal Stage 1 funding rule.

Proposition 4: The optimal capital allocation mechanism results in Stage 1 investment whereby the principal:

(i) Underinvests in project funding at Stage 1 relative to the First-Best solution.

(ii) Underinvests *less* in project funding at Stage 1 relative to the No Commitment model.

First-Best investing requires that the principal only fund projects at Stage 1 if the expected project profits are non-negative at time *t*=1; i.e., $z(c_i) > 0 \Leftrightarrow \sum_{j=1}^{k} \rho_j (R - m_j)^+ - c_i \geq 0$. Relative to First-Best, Proposition 4 finds that when the manager is privately informed as to his Stage 1 cost, the principal optimally distorts her Stage 1 investment in addition to the distortions described in Stage 2 (Proposition 3). However, the distortions across the two stages act as substitutes: by committing to Stage 2 distortions (relative to the No Commitment model), the principal mitigates the distortions necessary at Stage 1 to elicit truthful reporting.

The discrete nature of both the research costs, C, and implementation costs, M, prevent the model from admitting a closed-form solution, however we can further characterize the optimal contract by deriving a comparative static result. In solving for the optimal contract, recall that the principal chooses both the threshold research costs, c_i, and the contingent implementation probabilities, p_j. Holding fixed the number of funded research projects (h), the following proposition examines how the set of implementation probabilities $\{p_j\}$ varies with changes to any *one* project's potential profitability, $G_j = R - m_j$.

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[15] Trading off productive efficiency and deadweight loss, although not informational rent arises in the implicit contracting literature (e.g., MacLeod [2003] and Rajan, and Reichelstein [2009]).

Proposition 5: Increases (decreases) in the project profitability of potential project j, G_j, to $G_j^{'}$ such that $G_{j+1} < G_j^{'} < G_{j-1}$, will result in larger (smaller) optimal implementation probabilities, $p_i^{'*}$, relative to p_i^{*}, *for all* $i \le h$.

Proposition 5 suggests that as *any one* potential project's profitability increases, *all* project implementation probabilities will increase, even those which may be ex-post unprofitable. The result follows from the fact that the implementation probabilities serve two purposes: (1) to generate profits for the principal and (2) to insure the manager's incentive compatibility. The first effect causes the principal to raise the implementation probability for any project as it becomes more profitable, whereas the second effect causes all other probabilities to follow, including those associated with potentially unprofitable projects. This implies that subsidies which promote *one* of the firm's potential products will raise the likelihood of the firm investing in *all* its potential products. The finding only holds for small changes to project profitability (i.e., which do not disrupt the original ordering of G), because larger changes in expected profits also encourage the principal to raise the number of research proposals funded in Stage 1, which the present model cannot address.

IV. An Example

In this section, we consider a two-stage investment project, and illustrate how the optimal contract varies across three models: First-Best contracting, the No Commitment model, and our multi-stage model with commitment.

Assume that $R = 33.5$, $C = \{1.5, 3, 4.5, 6, 7.5\}$, $M = \{10, 20, 30, 35\}$, and $\rho_j \equiv \rho = \frac{1}{4}$. We first characterize the solution to the First-Best model and the optimal solution to the No Commitment model before solving for the optimal contract in our multi-stage setting with commitment.

In the First-Best setting, the principal observes the minimum cost required to initiate research, c_i, therefore she need only pay the manager his required investment; i.e., $b_i^{FB} = c_i$. The principal optimally funds all Stage 1 research with a non-negative expected value, $c_i \le \sum_{j=1}^{k} \rho_j \left(R - m_j\right)^{+}$. Given our example, the principal will only implement projects with continuation costs: m_1, m_2, m_3 implying that all funded Stage

1 projects have a $p^{FB} = p_i^{FB} = 0.75$ probability of implementation. In Table 1, we list all possible contracts; i.e., we consider contracts where the principal funds the least costly h managers for $h = 1,\ldots,5$ and the principal's resulting expected utility $U_{P,h}^{FB}$ for the different possible Stage 1 threshold costs Table 1 shows that the principal's expected utility is maximized by funding all Stage 1 projects regardless of the manager's Stage 1 cost.[16]

Table 1: First-Best Contracts

Most costly Stage 1 cost funded				
$c_{h=1} = 1.50$	$c_{h=2} = 3.00$	$c_{h=3} = 4.50$	$c_{h=4} = 6.00$	$c_{h=5} = 7.50$
Principal's resulting utility				
$U_{P,h=1}^{FB} = 1.73$	$U_{P,h=2}^{FB} = 3.15$	$U_{P,h=3}^{FB} = 4.58$	$U_{P,h=4}^{FB} = 5.10$	$U_{P,h=5}^{FB} = 5.63$

In the No Commitment model, the Stage 2 implementation rule is the same as in First-Best, $p^{NC} = p_i^{NC} = 0.75$. In order to accommodate constraint (IC), the optimal contract will allocate a single budget to the manager, regardless of his cost report, though the report can be used to decide whether or not to provide funding. The principal thus chooses a Stage 1 threshold cost, c_h and an optimal budget, $b_{i,h}^{NC} = c_h, \forall i \le h$, where $b_{i,h}^{NC}$ is the level of funding allocated to the manager upon reporting a cost c_i when the Stage 1 hurdle is set to c_h. Table 2 shows the principal's expected utility $U_{P,h}^{NC}$ for the different possible Stage 1 threshold costs.

Table 2: No Commitment Contracts

Most costly Stage 1 cost funded				
$c_{h=1} = 1.50$	$c_{h=2} = 3.00$	$c_{h=3} = 4.50$	$c_{h=4} = 6.00$	$c_{h=5} = 7.50$
Principal's utility				
$U_{P,h=1}^{NC} = 1.73$	$U_{P,h=2}^{NC} = 2.85$	$U_{P,h=3}^{NC} = 3.38$	$U_{P,h=4}^{NC} = 3.30$	$U_{P,h=5}^{NC} = 2.63$

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[16] The principal's preferred contract is highlighted in each table.

Due to the adverse selection problem, the principal now chooses a threshold ($c_{h=3}^{NC} = 4.5$) which is smaller than that in the First-Best setting ($c_{h=5}^{FB} = 7.5$).If the principal can commit to a two-stage contract, then according to Proposition 3, she will distort Stage 2 investments, and by Proposition 2, distort her choice of Stage 1 investments away from First-Best. By considering the alternative Stage 1 cost thresholds, we solve for the optimal continuation probabilities, Stage 1 budgets, Stage 2 thresholds and the principal's resulting expected utility in Table 3.

Table 3: Multi-Period Contracts With Commitment

Most costly Stage 1 cost funded

$c_{h=1} = 1.50$	$c_{h=2} = 3.00$	$c_{h=3} = 4.50$	$c_{h=4} = 6.00$	$c_{h=5} = 7.50$

Principal's utility

$U_{P,h=1} = 1.73$	$U_{P,h=2} = 2.86$	$U_{P,h=3} = 3.45$	$U_{P,h=4} = 3.55$	$U_{P,h=5} = 3.10$

Stage 1 budgets

$b_{1,1} = 1.50$	$b_{1,2} = 2.80$	$b_{1,3} = 3.38$	$b_{1,4} = 4.56$	$b_{1,5} = 5.91$
	$b_{2,2} = 3.00$	$b_{2,3} = 4.00$	$b_{2,4} = 5.37$	$b_{2,5} = 5.96$
		$b_{3,3} = 4.50$	$b_{3,4} = 5.50$	$b_{3,5} = 6.91$
			$b_{4,4} = 6.00$	$b_{4,5} = 7.07$
				$b_{5,5} = 7.50$

Stage 2 implementation probabilities

$p_1 = 0.75$	$p_1 = 0.87$	$p_1 = 1.00$	$p_1 = 1.00$	$p_1 = 1.00$
	$p_2 = 0.75$	$p_2 = 0.75$	$p_2 = 0.79$	$p_2 = 0.99$
		$p_3 = 0.50$	$p_3 = 0.75$	$p_3 = 0.75$
			$p_4 = 0.50$	$p_4 = 0.70$
				$p_5 = 0.50$

Stage 2 Threshold Continuation Costs

$\bar{m}_1 = 30$	$\bar{m}_1 = 35$	$\bar{m}_1 = 35$	$\bar{m}_1 = 35$	$\bar{m}_1 = 35$
	$\bar{m}_2 = 30$	$\bar{m}_2 = 30$	$\bar{m}_2 = 35$	$\bar{m}_2 = 35$
		$\bar{m}_3 = 20$	$\bar{m}_3 = 30$	$\bar{m}_3 = 30$
			$\bar{m}_4 = 20$	$\bar{m}_4 = 30$
				$\bar{m}_5 = 20$

Table 3 reveals several notable facts. First, in accordance with Proposition 2, for any given Stage 1 hurdle, the Stage 2 thresholds are weakly (in the example, strictly) decreasing in the manager's continuation cost observation. Furthermore, holding the reported Stage 1 cost fixed, the budgets and Stage 2 probabilities are both increasing in the Stage 1 hurdle, c_h. Second, the principal *strictly* overinvests in Stage 2 should the manager report either of the two smallest Stage 1 costs, since $R < \overline{m}_1, \overline{m}_2$. Third, as the Stage 1 hurdle, c_h, increases, the Stage 2 implementation probability for the most costly funded Stage 1 cost (i.e., c_h), weakly decreases, which further illustrates that the principal trades off Stage 1 and Stage 2 investment distortions. Finally, note that under the principal's preferred contract ($c_h = c_4$), she implements projects in Stage 2 with probability $p_{2,1} = p_{2,2} = p_{2,3} = 1$ and $p_{2,4} = 0.16$ for continuation costs m_1, m_2, m_3 and m_4 respectively. As suggested earlier, the principal stochastically implements the costliest (m_4) of all implemented projects. The intuition behind her randomization rests on the fact that a deterministic Stage 2 investment strategy is too coarse: requiring the principal to set $p_{2,4} = 1$ would provide the manager with excessive unrecoverable rents, whereas setting $p_{2,4} = 0$ would force the principal to forgo an otherwise profitable project. Put differently, the randomization does not constitute a mixed strategy whereby the principal is indifferent between $p_{2,4} = 0$ and $p_{2,4} = 1$, but instead results from the coarseness associated with the finite supports of both c and m.

V. Conclusion

Many capital investment decisions, especially those concerning R&D or new technologies (e.g. venture capital) involve multiple stages as the success of such investments is incrementally determined with the arrival of interim information. Such investments are particularly prone to problems of asymmetric information, as firms are unable to discern upfront costs at the outset. In this paper we examine how the optimal capital budgeting process changes as a result of the need to implement projects across multiple stages when firms can commit to their continuation decisions ex-ante. We base our analysis on a model in which the incentive problem arises because the manager privately observes the minimum required investment for the first (experiment) stage of a project and any excess funding in Stage 1 benefits the manager personally. We find that the optimal capital budgeting solution for the two-stage process is quite different from that for one-stage processes. The presence of a second stage funding decision provides the principal an additional lever of control with which to influence manager's revelation of private information in the first stage. In a one-stage game, the principal finds it optimal to either fund a project at a fixed level or not to fund it at all. In the presence of a second-stage funding decision, the principal finds it optimal to vary the amount of Stage 1 funding based on the manager's report. Further, the principal also finds it optimal to distort her Stage 2 implementations to control the manager's Stage 1 truth-telling incentives. In particular, the principal finds it optimal to both overinvest and underinvest, relative to the sequentially rational investment level, depending on the manager's Stage 1 observation. Our model thus provides an additional explanation for the empirical evidence suggesting that firms both under- and overinvest internally. On average, we find that the firm optimally underinvests in Stage 2 and Stage 1, akin to the optimal contract obtained in the single-stage literature. Though the additional control provided by the second stage allows the principal to reduce the level of underinvestment at Stage 1 relative to the single-stage literature, which in turn supports our primary conclusion that the principal treats the distortions at Stages 1 and 2 as substitutes.

Our model assumes that the interim information available at Stage 2 is independent of the manager's Stage 1 cost. If instead, the manager's cost were correlated with the Stage 2 information, then the principal could further reduce the manager's informational rents by exploiting the statistical relation. However, to the extent that the Stage 2 information does not *perfectly* reveal the manager's cost, then the same tensions examined in this study continue to hold. It would be interesting to see whether a statistical dependence between the two pieces of information mitigates or exacerbates the distortions identified in this study, though we suspect this will largely depend on the particular distributions examined. Another avenue for extending the present work could consider a privately informed principal, whereby the cost of continuing the project in Stage 2, m_j, is not entirely public information. In such a setting, the principal can no longer perfectly commit to particular Stage 2 investment rules, though it is unclear how this will impact the optimal contract. However, to the extent that she can partially commit to alternate investment rules, then we believe our present results will continue to influence the optimal contract.

VI. Appendix

Summary of in-text notation:

c_i	Manager's Stage 1 cost from support $C = \{c_1, \ldots, c_n\}$
δ	Difference in Stage 1 costs; i.e., $c_i - c_{i-1}$ for $i > 1$
$\hat{c}_i$	Manager's reported cost
R	Principal's *t*=5 revenue following project implementation in Stage 2
m_j	Project Stage 2 continuation cost, from support $M = \{m_1, \ldots, m_k\}$
r	Largest index such that $m_r \le R$
$G_j = R - m_j$	Principal's *t*=5 profits following implementation of a project with continuation cost, m_j, in Stage 2
ρ_j	Probability of Stage 2 continuation cost m_j; i.e., $f\left(m_j\right)$
$z\left(c_j\right)$	Probability of principal funding project of reported cost c_j
h	Largest index such that $z\left(c_h\right) > 0$
$b_i \equiv b\left(c_i\right)$	Level of funding allocated to manager with reported cost c_i
$p_{i,j}$	Probability of a project with continuation cost m_j being implemented in Stage 2 following a reported Stage 1 cost of c_i
$p_i \equiv \sum_j^k \rho_j p_{i,j}$	Probability of *any* project being implemented in Stage 2 following a reported Stage 1 cost of c_i
$\overline{m}_i$	Threshold Stage 2 cost following a reported *Stage 1* cost of c_i, such that all projects with continuation cost in excess of $\overline{m}_i$ are aborted in Stage 2
$b_{i,\eta}$ *(section IV only)*	Stage 1 funding for manager with reported cost c_i and $h = \eta$
$U_{i,\eta}^{FB}$ (*section IV only*)	Principal's expected utility under First-Best with reported cost c_i and $h = \eta$
$U_{i,\eta}^{NC}$ (*section IV only*)	Principal's expected utility under No Commitment model with reported cost c_i and $h = \eta$
$U_{i,\eta}$ (*section IV only*)	Principal's expected utility under multi-stage commitment model with reported cost c_i and $h = \eta$

Proofs

Proposition 1: (i) Note that the manager's utility does not depend on $p_{i,j}$ *per-se*, but only on the total implementation probability $p_i = \sum_{j=1}^{k} \rho_j p_{i,j}$, therefore in evaluating the manager's expected utility, it suffices to consider the total implementation probabilities, p_i, instead of the individual probabilities, $p_{i,j}$. Thus, we can write the expected utility of a manager who observes c_i as $z_i p_i \left(b_i - c_i\right)$. We first establish that if $z_i p_i > 0$ then $b_{i-1} \le b_i$ with $p_i = \sum_{j=1}^{k} \rho_j p_{i,j}$, which will allow us to establish that $z_i \in \left\{0,1\right\}$ and that the optimal contract employs a threshold Stage 1 funding rule.

Adding the (IC) constraints for i and $i-1$, yields: $z_{i-1} p_{i-1}(c_i - c_{i-1}) \ge z_i p_i (c_i - c_{i-1})$ or $z_i p_i \le z_{i-1} p_{i-1}$. To satisfy (IC) for *i*, we must have: $z_i p_i (b_i - c_i) \ge z_{i-1} p_{i-1} (b_{i-1} - c_i)$, thus $\left(b_i - c_i\right) \ge \left(b_{i-1} - c_i\right)$, or equivalently, $b_i \ge b_{i-1}$.

Next, we claim that projects are funded with probability 1 or 0; i.e., $z_i \in \left\{0,1\right\}$. To see this, note that any $z_i p_i \in \left(0,1\right)$ can be implemented in the manager's constraints with $z_i' = 1$ and $p_i' = z_i p_i$, hence the constraints are unaffected by restricting z_i to $\{0,1\}$. Additionally, note that the principal's objective function is linear in z_i, thus the optimal solution z_i^* must belong to the boundary $\{0,1\}$.

We finalize the proof to Proposition 1 (i) by showing that a threshold research cost, c_h, exists, for which all less costly research is funded, and all costlier research is rejected. Suppose c_i is funded, then $z_i = 1$, and hence $p_i > 0$, for otherwise the allocated budget is wasted. Since $z_i p_i$ was shown to decrease in *i*, we have $z_{i-1} p_{i-1} > 0$, therefore if a project is funded, all lower cost projects also receive funding. This allows us to unambiguously denote the principal's objective function when h managers receive funding as $U^h\left(P\right)$.

To prove (ii), we begin by showing that the manager's expected rent is strictly decreasing in his research cost realization, c_i. We then prove that the upward (IC) constraints must bind in equilibrium, which enables us to derive the optimal funding rule. We finalize the proof by showing that the downward (IC) constraints are satisfied within this construct.

From (IC) for any i, l such that $p_i, p_l > 0$ and $c_i < c_l$ we have: $p_i(b_i - c_i) \ge p_l(b_l - c_i) > p_l(b_l - c_l)$. That is, the manager's expected rent is decreasing in his cost observation. If c_h is the greatest research cost observation with $p_h > 0$, then it must be that $b_h = c_h$; i.e., when the manager truthfully reports the largest allowable research cost, he earns no expected rents. If not, then one could decrease all b_j for $j < k$ by $b_h - c_h > 0$ without violating (IR), since we have already shown that if $p_i > 0$, then $b_i - c_i > b_h - c_h$. Further, since all budgets are decreased equally, the IC constraints are unaffected.

We now show that only the adjacent upward (IC) constraints need to bind. First, it is straight-forward to establish that satisfying the adjacent upward constraints implies that the other upward constraints are satisfied, therefore, we restrict our attention to adjacent constraints. If $p_i > 0$, then if the upward (IC) constraint binds, we must have $p_i(b_i - c_i) = p_{i+1}(b_{i+1} - c_i)$ or equivalently, $b_i = \frac{p_{i+1}}{p_i}\left(b_{i+1} - c_i\left(1 - \frac{p_i}{p_{i+1}}\right)\right)$.

Solving iteratively, we obtain: $b_i = c_i + \delta \sum_{q=i+1}^{h} \frac{p_q}{p_i}$ as was to be shown in part (ii). To see that the adjacent upward (IC) constraints must bind, suppose that the (IC) constraint for a manager reporting c_i, does not bind; i.e., $p_i\left(b_i - c_i\right) - p_{i+1}\left(b_{i+1} - c_i\right) = \varepsilon > 0$. Consider a change wherein we reduce the budget facing all managers with cost c_i or less by γ such that the adjacent upward (IC) constraint of a manager who observed c_i binds. The change does not affect the adjacent upward (IC) constraints of all manager's with cost less than c_i, and their (IR) constraints are satisfied as long as manager *i*'s (IR) is satisfied (recall that the expected rents are decreasing in type). The (IR) constraint for manager *i*+1 assures that his utility is at least 0, therefore $p_i\left(b_i - c_i\right) - \varepsilon = p_{i+1}\left(b_{i+1} - c_i\right) > p_{i+1}\left(b_{i+1} - c_{i+1}\right) \geq 0$, hence the new contract continues to satisfy the (IR) constraint of a manager who observed c_i. Because this manager is now allocated a smaller budget, the downward (IC) constraint which insures that a manager with reported cost greater than c_i does not report a cost of c_i, is strengthened. The contract modification generates additional profit, therefore the upward (IC) constraints must bind.

Finally, the downward (IC) constraints are satisfied as a result of the decreasing implementation probabilities (previously shown) as:

$$\begin{aligned}
&p_{i+1}\left(b_{i+1} - c_{i+1}\right) \geq p_i\left(b_i - c_{i+1}\right) \\
&\qquad \Rightarrow p_{i+1}\left(b_{i+1} - c_{i+1}\right) \geq p_{i+1}\left(b_{i+1} - c_i\left(1 - \frac{p_i}{p_{i+1}}\right)\right) - p_i c_{i+1} \\
&\qquad \Rightarrow p_i\left(c_{i+1} - c_i\right) \geq p_{i+1}\left(c_{i+1} - c_i\right).
\end{aligned}$$

Proposition 2: We first solve the principal's maximization problem when h managers receive Stage 1 funding using only first order conditions and prove that the ensuing solution satisfies the proposition. Afterwards, we show that the first order conditions are sufficient to solve the principal's problem. From the proof to Proposition 1, we can restate the principal's problem when h managers receive Stage 1 funding:

$$U^h = \max_{p_{i,j}} \frac{1}{n} \sum_{i=1}^{h} \sum_{j=1}^{k} \left(\rho_j p_{i,j}\left(R - m_j\right) - c_i - \frac{\sum_{s=i+1}^{h} p_s}{p_i} \delta \right).$$

Taking the derivative with respect to $p_{i,j}$ yields:

$$\frac{\partial U^h}{\partial p_{i,j}} = \frac{\rho_j}{n}\left[R - m_j + \sum_{s=i+1}^{h} \frac{p_s}{p_i^2}\delta - \sum_{t=1}^{i-1} \frac{1}{p_t}\delta \right].$$

The principal will optimally set $p_{i,j}$ such that $\frac{\partial U^h}{\partial p_{i,j}} = 0$. If equality cannot be met with $p_{i,j} \in [0,1]$, then the principal sets $p_{i,j} = 1$ if $\frac{1}{\delta}(R - m_j) > \sum_{t=1}^{i-1}\frac{1}{p_t} - \sum_{s=i+1}^{h}\frac{p_s}{p_i^2}$, and $p_{i,j} = 0$ if $\frac{1}{\delta}(R - m_j) < \sum_{t=1}^{i-1}\frac{1}{p_t} - \sum_{s=i+1}^{h}\frac{p_s}{p_i^2}$. Since $\frac{1}{\delta}(R - m_j)$ is decreasing in m_j, the principal will set $p_{i,j} = 1$ for $m_j \le \bar{m}_{i-1}$, $p_{i,j} \in (0,1]$ for $m_j = \bar{m}_i$, and $p_{i,j} = 0$ for $m_j > \bar{m}_i$, which proves part (i).

To see part (ii), first note that part (i) implies that $p_{i,j}$ is weakly decreasing in m_j. Next, recall that Proposition 1 found $c_s \ge c_i \Leftrightarrow b_s \ge b_i$. Using constraint (IC) for manager *i*, we have:

$\sum_{j=1}^{k} \rho_j p_{i,j}(b_i - c_i) \ge \sum_{j=1}^{k} \rho_j p_{s,j}(b_s - c_i) \Rightarrow \sum_{j=1}^{k} \rho_j p_{i,j} \ge \sum_{j=1}^{k} \rho_j p_{s,j}$, thus $p_{i,j}$ is weakly decreasing in c_i, as was to be shown.

To show that the first-order approach yields a local maximum, we first reformulate the problem when h managers receive Stage 1 funding to facilitate notation. Similar to above, we denote $p_i \equiv \sum_{j=1}^{k} \rho_j p_{i,j}$. To see that the principal's problem is identical whether she solves for p_i or $p_{i,j}$, note that for any vector, $P = \{p_1, \ldots, p_h\}$ she will optimally set:

$$p_{i,j} = \begin{cases} 1 & \text{if } \sum_{s=1}^{j} \rho_s \le p_i \\ \dfrac{p_i - \sum_{s=1}^{j-1}\rho_s}{\rho_i} & \text{if } \sum_{s=1}^{j} \rho_s > p_i > \sum_{s=1}^{j-1}\rho_s \\ 0 & \text{if } \sum_{s=1}^{j-1} \rho_s > p_i \end{cases}$$

Thus, it is sufficient to show if the principal selects a solution vector $P^* = \{p_1^*, \ldots, p_h^*\} \in [0,1]^h$ according to the first-order conditions, then that solution constitutes a local maximum in order to prove that the original $h \times k$ solution generated via the rules above is itself a local maximum to original problem. We summarize our notation below:

$\vert P \vert$	Dimensionality of vector P
$\gamma(p)$	$G_j p$ when $\sum_{i=1}^{j-1}\rho_i < p \leq \sum_{i=1}^{j}\rho_i$
$\gamma'(p)$	G_j when $\sum_{i=1}^{j-1}\rho_i < p \leq \sum_{i=1}^{j}\rho_i$
$\gamma''(p)$	Change in project payoff at p
$d(P,\varepsilon)$	Open ball of radius $\vert \varepsilon \vert$ in $[0,1]^{\vert P \vert}$ centered at P
P^*	Proposed solution vector with i^{th} entry $0 \leq p_i^* \leq 1$
P_{-i}	Arbitrary vector in $[0,1]^{h-1}$
P_{-i}^*	Solution vector P^* with i^{th} entry omitted
$P_{-i}(p_i, d(P_{-i}^*,\varepsilon))$	Arbitrary vector from set $\arg\max_{s\in d(P_{-i}^*,\varepsilon)} U^h(s,p_i)$
h	Assumed highest funded first-stage cost project
B	Set of indices i where p_i^* satisfies FOC with equality (defined later)
NB	Set of indices i where p_i^* does not satisfy FOC with equality
$\vert B \vert$	Cardinality of set B (number of elements contained)
P_B^*	Sub-vector consisting exclusively of $p_i^* \in P^*$ when $i \in B$
P_{NB}^*	Sub-vector consisting exclusively of $p_i^* \in P^*$ when $i \in NB$

With this new notation, for a fixed h the principal solves $\max_P f(P)$, with:

$$f(P) = \frac{1}{n}\sum_{i=1}^{h}\left(\gamma(p_i) - c_i - \frac{\delta}{p_i}\sum_{t=i+1}^{h} p_t\right).$$

We label $U^h(P)$ the principal's payoff when the largest Stage 1 cost observation to receive funding is c_h and Stage 2 projects are implemented with probability P. The function $\gamma(p_i)$ in $f(P)$ is piece-wise linear, concave and maps $[0,1] \to [0,\infty)$ such that:

$$\gamma(p_i) = \begin{cases} p_i G_1 & \text{for } p_i \leq \rho_i \\ \rho_1 G_1 + (p_i - \rho_1) G_2 & \text{for } \rho_1 < p_i \leq \rho_1 + \rho_2 \\ \vdots & \vdots \\ \sum_{s=1}^{k-1}\rho_s G_s + \left(p_i - \sum_{r=1}^{k-1}\rho_r\right) G_h & \text{for } \sum_{s=1}^{k-1}\rho_s < p_i \leq \sum_{s=1}^{k}\rho_s \end{cases}$$

By virtue of being a concave and piece-wise linear, the first and second derivative of γ with respect to p_i, are given by:

$$\gamma'(p_i) = \begin{cases} G_j & \text{for } \sum_{s=1}^{j-1}\rho_s < p_i < \sum_{s=1}^{j}\rho_s \\ G_j & \text{for } p_i \to \left(\sum_{s=1}^{j}\rho_s\right)_- \\ G_{j+1} & \text{for } p_i \to \left(\sum_{s=1}^{j}\rho_s\right)_+ \end{cases} \text{, and}$$

$$\gamma''(p_i) = \begin{cases} 0 & \text{for } p_i \neq \sum_{s=1}^{j}\rho_s \quad \text{and arbitrary } j \leq k \\ G_j - G_{j-1} < 0 & \text{for } p_i = \sum_{s=1}^{j}\rho_s \quad \text{and arbitrary } j \leq k. \end{cases}$$

The constant payoffs, $G_j = R - m_j$, are bounded, which in turn bounds both the first and second derivative of γ. We label the costliest Stage 2 continuation project the principal will implement with positive probability after funding Stage 1 project c_i, w_i; that is, if $\sum_{s=1}^{j-1}\rho_s < p_i \leq \sum_{s=1}^{j}\rho_s$ then the Stage 2 revenue of w_i is defined by $\lim_{p \to p_{i_-}} \gamma'(p) = G_{w_i}$. Put differently, if $\bar{m}_i = m_j$, then $j = w_i$.

We restrict our attention to solution vectors, P^*, whereby $p_h^* > 0$ and p_i^* is decreasing in i without loss of generality. If the proposed solution has $p_h^* = 0$, then by construction, a smaller value of h will generate greater profits. Furthermore, the implementation probabilities, p_i, must be decreasing in i in order for incentive compatibility to be satisfied, which in turn implies that G_{w_i} is decreasing in i.

If $\frac{\partial f(p_i^*)}{\partial p_i} = 0$ we say that the first-order condition (FOC) for p_i binds, and hence $i \in B$, whereas if $\frac{\partial f(p_i^*)}{\partial p_i} \neq 0$ we say that the first-order constraint for p_i does not bind, and hence $i \in NB$. Note that $NB \cap B = \varnothing$.

Consider a perturbation, $|\varepsilon| > 0$ to p_i^*. To show that the principal can only decrease her profits with the perturbation, we must show that: $\max_{P_{-i}} f\left(P_{-i}, p_i^* + \varepsilon\right) < f\left(P^*\right)$; i.e., the best the principal can do with $p_i = p_i^* + \varepsilon$ is strictly less than her payoff when $p_i = p_i^*$.

Case 1: $i \in NB$

If $i \in NB$, then it must be the case that $p_i^* = \sum_{s=1}^{j}\rho_s$ for some $j \leq k$, $\lim_{p_i \to \left(p_i^*\right)_-} \frac{\partial f(P^*)}{\partial p_i} > 0$, and $\lim_{p_i \to \left(p_i^*\right)_+} \frac{\partial f(P^*)}{\partial p_i} < 0$ therefore $\frac{\partial f(P_{-i}^*, p_i^* + \varepsilon)}{\partial p_i} < 0$ for sufficiently small $\varepsilon > 0$, as long as $p_i^* \neq 1$. Thus, suppose for the time being that $p_i^* \neq 1$. By the continuity of $\left.\frac{\partial f(P_{-i}^*, p_i)}{\partial p_i}\right|_{p_i \to p_{i-}^*}$ and $\left.\frac{\partial f(P_{-i}^*, p_i)}{\partial p_i}\right|_{p_i \to p_{i+}^*}$ in P_{-i}, we can choose an $\varepsilon > 0$ sufficiently small such that

$$\frac{\partial f(P_{-i}, \underline{p_i^*})}{\partial p_i} > 0 > \frac{\partial f(P_{-i}, \overline{p_i^*})}{\partial p_i} \tag{1}$$

for $P_{-i} \in d(P_{-i}^*, \varepsilon) \subset [0,1]^{h-1}$ and $p_i^* - \varepsilon < \underline{p_i^*} < p_i^* < \overline{p_i^*} < p_i^* + \varepsilon$. By the mean value theorem:

$$\begin{aligned}\max_{P_{-i} \in d(P_{-i}^*, \varepsilon)} f(P_{-i}, p_i^* + \varepsilon) &\equiv f(P_{-i}(p_i^* + \varepsilon, d(P_{-i}^*, \varepsilon)), p_i^* + \varepsilon) \\ &= f(P_{-i}(p_i^* + \varepsilon, d(P_{-i}^*, \varepsilon)), p_i^*) + \varepsilon \frac{\partial f(P_{-i}(p_i^* + \varepsilon, d(P_{-i}^*, \varepsilon)), \tau)}{\partial p_i},\end{aligned}$$

where $\tau \in (p_i^*, p_i^* + \varepsilon)$. From the RHS of (1) it follows that $\frac{\partial f(P_{-i}(p_i^* + \varepsilon, d(P_{-i}^*, \varepsilon)), \tau)}{\partial p_i} < 0$, implying that $\max_{P_{-i} \in d(P_{-i}^*, \varepsilon)} f(P_{-i}, p_i^* + \varepsilon) < f(P_{-i}\left(p_i^* + \varepsilon, d\left(P_{-i}^*, \varepsilon\right)\right), p_i^*)$. By definition of h, $p_i^* > 0$, therefore the same argument shows that

$$\max_{P_{-i} \in d(P_{-i}^*, \varepsilon)} f\left(P_{-i}, p_i^* - \varepsilon\right) < f\left(P_{-i}(p_i^* - \varepsilon, d(P_{-i}^*, \varepsilon)), p_i^*\right),$$

implying $\max_{P_{-i} \in d(P_{-i}^*, \varepsilon)} f(P_{-i}, p_i) < f(P^*)$, as was to be shown.

If $p_i^* = 1$ and $i \in NB$, then the only possible permutation is given by $p_i^* - \varepsilon$ where $\varepsilon > 0$, and again, $\left.\frac{\partial f(P_{-i}, p_i)}{\partial p_i}\right|_{p_i \to (1)_-} > 0$. By the continuity of $\left.\frac{\partial f(P_{-i}^*, p_i)}{\partial p_i}\right|_{p_i \to (1)_-}$, we can choose an $\varepsilon > 0$ sufficiently small such that $\frac{\partial f(P_{-i}, \underline{p_i^*})}{\partial p_i} > 0$ for $P_{-i} \in d(P_{-i}^*, \varepsilon)$ and $p_i^* - \varepsilon < \underline{p_i^*} < p_i^*$. By the mean value theorem, we can write:

$$\begin{aligned}\max_{P_{-i} \in d(P_{-i}^*, \varepsilon)} f(P_{-i}, p_i^* - \varepsilon) &= f(P_{-i}(p_i^* - \varepsilon, d(P_{-i}^*, \varepsilon)), p_i^* - \varepsilon) \\ &= f(P_{-i}(p_i^* - \varepsilon, P_{-i}^*, \varepsilon), p_i^*) - \varepsilon \frac{\partial f(P_{-i}(p_i^* - \varepsilon, d(P_{-i}^*, \varepsilon)), \tau)}{\partial p_i},\end{aligned}$$

where $\tau \in (p_i^* - \varepsilon, p_i^*)$, hence $\frac{\partial f(P_{-i}(p_i^* - \varepsilon, d(P_{-i}^*, \varepsilon)), \tau)}{\partial p_i} > 0$, implying that

$\max_{P_{-i} \in d(P_{-i}^*, \varepsilon)} f(P_{-i}, p_i^* - \varepsilon) < f(P_{-i}(p_i^* - \varepsilon, d(P_{-i}^*, \varepsilon)), p_i^*)$, as was to be shown.

Since the only assumption made on i was that $i \in NB$, the same argument applies to all elements of NB; in particular, for any $s \in NB$, the function $f(P)$ restricted to the domain $P \in d(P^*, \varepsilon)$, is maximized if $p_s = p_s^*$. Thus, if $f(P) \geq f(P^*)$ with $P \in d(P^*, \varepsilon) \subset [0,1]^h$, it must be the case that $f(P_B, P_{NB}^*) \geq f(P_B^*, P_{NB}^*)$. Therefore to complete the proof, we must prove that $f(P_B, P_{NB})\big|_{P_{NB}=P_{NB}^*}$ is maximized at P_B^* as shown below.

Case 2: $i \in B$

If $i \in B$, then $\dfrac{\partial f(P^*)}{\partial p_i} = 0$. Moreover, $\dfrac{\partial^2 f(P^*)}{\partial p_i \partial p_i} = \dfrac{1}{n}\left(-\dfrac{2}{(p_i^*)^3} \sum_{t=i+1}^{h} p_t + \gamma''(p_i^*) \right)$. Since $p_t^* \geq p_h^* > 0$, and $\gamma''(p_i^*) \leq 0$, $\dfrac{\partial^2 f(P^*)}{\partial p_i \partial p_i}$ is trivially negative for $i < h$. If, on the other hand, $i = h$, then $\dfrac{\partial^2 f(P^*)}{\partial p_h \partial p_h} = \gamma''(p_h^*) \leq 0$. Suppose $\dfrac{\partial^2 f(P^*)}{\partial p_i \partial p_i} < 0$, we can choose a sufficiently small perturbation, $\varepsilon > 0$ such that $\dfrac{\partial f(P_{-i}^*, \overline{p_i^*})}{\partial p_i} < \dfrac{\partial f(P^*)}{\partial p_i} = 0 < \dfrac{\partial f(P_{-i}^*, \underline{p_i^*})}{\partial p_i}$ for $P_{-i} \in d(P_{-i}^*, \varepsilon) \subset [0,1]^{h-1}$ and $p_i^* - \varepsilon < \underline{p_i^*} < p_i^* < \overline{p_i^*} < p_i^* + \varepsilon$. By the mean value theorem:

$$\begin{aligned} \max_{P_{-i} \in d(P_{-i}^*, \varepsilon)} f(P_{-i}, p_i^* + \varepsilon) &= f(P_{-i}(p_i^* + \varepsilon, d(P_{-i}^*, \varepsilon)), p_i^* + \varepsilon) \\ &= f(P_{-i}(p_i^* + \varepsilon, d(P_{-i}^*, \varepsilon)), p_i^*) + \varepsilon \frac{\partial f(P_{-i}(p_i^* + \varepsilon, d(P_{-i}^*, \varepsilon)), \tau)}{\partial p_i}, \end{aligned}$$

where $\tau \in (p_i^*, p_i^* + \varepsilon)$. Thus, $\dfrac{\partial f(P_{-i}(p_i^* + \varepsilon, d(P_{-i}^*, \varepsilon)), \tau)}{\partial p_i} < 0$, implying that $\max_{P_{-i} \in d(P_{-i}^*, \varepsilon)} f(P_{-i}, p_i^* + \varepsilon) < f(P_{-i}(p_i^* + \varepsilon, d(P_{-i}^*, \varepsilon)), p_i^*)$.

On the other hand, if $i = h$ and $\dfrac{\partial^2 f(P^*)}{\partial p_h \partial p_h} = 0$, then any additional small perturbation $\varepsilon > 0$, will leave the principal's profits unchanged, as her maximization problem is piece-wise linear in p_h, which completes the proof.

Proposition 3:

To prove part (i), note that the first derivative of the principal's expected utility with respect to p_1 is $\dfrac{\partial f(P)}{\partial p_1} = \dfrac{1}{n}\left(\gamma'(p_1^*) + \dfrac{\delta}{\left(p_1^*\right)^2} \sum_{t=2}^{h} p_t^* \right) \geq 0$, which implies $\gamma'(p_1^*) \leq 0$ whereas the efficient investment requires $\gamma'\left(p_1^{FB}\right)\geq 0$. Because γ is concave, and $\frac{\partial^2 f\left(P\right)}{\partial p_i \partial p_j}>0$ for $i\neq j$, the principal will invest weakly more in the second-best setting than she would in the first-best setting. If $R\geq m_k$, overinvestment is not possible and the principal will invest ex-post efficiently; i.e., set $p_1=1$. Since the probabilities, p_i^*, are ordered, if overinvestment takes place with *i* = 1, then the principal may also overinvest for higher Stage 1 cost realizations. To prove (ii); i.e., that the principal does not overinvest in response to all Stage 1 cost reports, note that the FOC with respect to p_h is: $\frac{\partial f\left(P\right)}{\partial p_h}=\frac{1}{n}\left(\gamma'(p_h^*)-\delta\sum_{t=1}^{h-1}\frac{1}{p_t^*}\right)\geq 0$, which implies $\gamma'(p_h^*)\geq 0$; i.e., underinvestment relative to First-Best when the manager reports sufficiently large cost realizations.

To prove part (iii): the FOC with respect to p_i is:

$$\frac{\partial f\left(P\right)}{\partial p_i}=\gamma'\left(p_i^*\right)-\delta\sum_{s=1}^{i-1}\frac{1}{p_s^*}+\delta\sum_{t=i+1}^{h}\frac{p_t^*}{\left(p_i^*\right)^2}\geq 0 .$$

Summing over all FOC (and substituting $\gamma'\left(p_i^*\right)=G_{w_i}$) yields:

$$\sum_{i=1}^{h}\left(G_{w_i}-\delta\sum_{s=1}^{i-1}\frac{1}{p_s^*}+\delta\sum_{t=i+1}^{h}\frac{p_t^*}{\left(p_i^*\right)^2}\right)\geq 0 ,$$

which can be rewritten as

$$\sum_{i=1}^{h}G_{w_i}-\delta\left(\sum_{i=1}^{h}\frac{\left(h-i\right)p_i^*-\sum\limits_{t=i+1}^{h}p_t^*}{\left(p_i^*\right)^2}\right)\geq 0 .$$

Since p_i^* is weakly decreasing in c_i, the term in brackets is non-negative, therefore $\sum_{i=1}^{h}G_{w_i}\geq 0$, and hence on average, there is weak underinvestment at Stage 2; i.e. $\sum_{i=1}^{h}\frac{G_{w_i}}{h}\geq 0$.

Proposition 4:

Define c_{FB} as the optimal Stage 1 threshold so that $\gamma\left(p_{FB}^{\dagger}\right)-c_{FB}\geq 0$ and $\gamma\left(p_{FB}^{\dagger}\right)-c_{FB+1}<0$, where $p_{FB}^{\dagger}$ is the project implementation rule under first-best (implement all $m_j\leq R$).

In our setting with commitment, consider an exogenously imposed number of Stage 1 funded managers, $h=FB+1$, such that $c_h=c_{FB+1}$, and let the solution therein be denoted by

$P^*(FB+1)=\left\{p_1^*\left(FB+1\right),p_2^*\left(FB+1\right),\ldots,p_{FB+1}^*\left(FB+1\right)\right\}$. In this case, we denote the principal's expected utility: $U^{FB+1}\left(P^*\left(FB+1\right)\right)$, where the superscript indicates the first stage implementation rule: $z_i^*=1\Leftrightarrow i\leq FB+1$ and $z_i^*=0\Leftrightarrow i>FB+1$. Thus:

$$U^{FB+1}P^*\left(FB+1\right)=\frac{1}{n}\sum_{i=1}^{FB+1}\left(\gamma\left(p_i^*\left(FB+1\right)\right)-c_i-\frac{\delta}{p_i^*\left(FB+1\right)}\sum_{s=i+1}^{FB+1}p_s^*\left(FB+1\right)\right).$$

Now, consider a change such that the principal does not fund projects with a reported Stage 1 cost of c_{FB+1} but keeps the implementation probabilities as defined earlier:

$$P^*(FB+1)=\left\{p_1^*\left(FB+1\right),p_2^*\left(FB+1\right),\ldots,p_{FB}^*\left(FB+1\right)\right\}.$$

Denote the principal's respective payoff therein, $U^{FB}P^*\left(FB+1\right)$. In particular:

$$U^{FB}\left(P^*\left(FB+1\right)\right)=\frac{1}{n}\sum_{i=1}^{FB}\left(\gamma\left(p_i^*\left(FB+1\right)\right)-c_i-\frac{\delta}{p_i^*\left(FB+1\right)}\sum_{s=i+1}^{FB+1}p_s^*\left(FB+1\right)\right).$$

The marginal benefit attributed to the decreased number of funded projects is given by (we omit the (FB+1) notation below as all implementation probabilities below are from $P^*(FB+1)=\left\{p_1^*\left(FB+1\right),p_2^*\left(FB+1\right),\ldots,p_{FB+1}^*\left(FB+1\right)\right\}$):

$$\begin{aligned}U^{FB}-U^{FB+1}&=\frac{1}{n}\sum_{i=1}^{FB}\left(\gamma\left(p_i^*\right)-c_i-\frac{\delta}{p_i^*}\sum_{s=i+1}^{FB}p_s^*\right)-\frac{1}{n}\sum_{i=1}^{FB+1}\left(\gamma\left(p_i^*\right)-c_i-\frac{\delta}{p_i^*}\sum_{s=i+1}^{FB+1}p_s^*\right)\\&=\frac{1}{n}\sum_{i=1}^{FB}\left(\gamma\left(p_i^*\right)-c_i-\frac{\delta}{p_i^*}\sum_{s=i+1}^{FB}p_s^*\right)-\frac{1}{n}\sum_{i=1}^{FB}\left(\gamma\left(p_i^*\right)-c_i-\frac{\delta}{p_i^*}\sum_{s=i+1}^{FB}p_s^*\right)\\&\quad-\frac{1}{n}\left(\gamma\left(p_{FB+1}^*\right)-c_{FB+1}-\sum_{i=1}^{FB}\frac{\delta p_{FB+1}^*}{p_i^*}\right)\\&=-\frac{1}{n}\left(\gamma\left(p_{FB+1}^*\right)-c_{FB+1}-\sum_{i=1}^{FB}\frac{\delta p_{FB+1}^*}{p_i^*}\right).\end{aligned}$$

As c_{FB} is defined as the highest c_i such that $\left(\max_p\gamma\left(p\right)\right)-c_i\geq 0$, it follows that $\gamma\left(p_{FB+1}^*\right)-c_{FB+1}<0$, hence the difference $U^{FB}\left(P^*\left(FB+1\right)\right)-U^{FB+1}\left(P^*\left(FB+1\right)\right)$ is positive and the principal never overinvests in Stage 1 relative to First-Best.

Next consider the principal's problem with commitment with an exogenously imposed number of Stage 1 funded managers, $h=FB$ such that $c_h=c_{FB}$. In this case the optimal Stage 2 implementation probabilities are denoted by $P^*(FB)=\left\{p_1^*,p_2^*,\ldots,p_{FB}^*\right\}\in\left[0,1\right]^{FB}$. Denote the Stage 2 implementation probabilities under First-Best $P^{\dagger}(FB)=\left\{p^{\dagger}_{FB},\ldots,p^{\dagger}_{FB}\right\}\in\left[0,1\right]^{FB}$. Because $h=FB$ is optimal in the First-Best setting, it must be the case that $U^{FB-1}\left(P^{\dagger}(FB)\right)-U^{FB}\left(P^{\dagger}(FB)\right)<0$, which implies that the loss in potential profits attributed to the omission of the manager reporting c_{FB}, is positive ($-\frac{1}{n}\left(\gamma\left(p^{\dagger}_{FB}\right)-c_{FB}\right)<0$). Recall that part (ii) of Proposition 3 showed that $p^{*}_{FB}\left(FB\right)\le p^{\dagger}_{FB}$, hence $\gamma\left(p^{*}_{FB}\left(FB\right)\right)-c_{FB}\le\gamma\left(p^{\dagger}_{FB}\right)-c_{FB}$, which implies: $U^{FB-1}\left(P^{\dagger}(FB)\right)-U^{FB}\left(P^{\dagger}(FB)\right)\le U^{FB-1}\left(P^{*}(FB)\right)-U^{FB}\left(P^{*}(FB)\right)$. Thus, the cost of underfunding at Stage 1 is always greater in the First-Best setting relative to that in the second-best setting, which implies that the principal weakly underinvest at Stage 1 in the second best setting.

To prove part (ii), we will show that the optimal number of funded cost observations in the No Commitment model (NC) is weakly less than that in the current paradigm. Let $\left\{P^{*}(h),B^{*}(h)\right\}=\left\{\left\{p_{1}^{*}(h),\ldots,p_{h}^{*}(h)\right\},\left\{b_{1}^{*}(h),\ldots,b_{h}^{*}(h)\right\}\right\}$ denote a solution to the principal's maximization problem with commitment: $\frac{1}{n}\max_{P,B}f(P,B;h)$, (note that we include the budget vector B as an argument because the principal may no longer employ the endogenously determined optimal budget in what follows) where the $P^{*}\left(h\right)$ notation comes from part (i), $f(P,B;h)=\frac{1}{n}\sum_{s=1}^{h}E[\gamma(p)\mid p=p_{s}]-b_{s}$, and the implementation probability notation is identical to that used in Proposition 2; i.e., $p_{i}^{*}\left(h\right)\in[0,1]$ is the probability of implementing a project pursuant to a reported cost of $c_{i}\le c_{h}$, when the principal chooses Stage 1 threshold h. By definition, c_{h} is the most costly funded research cost under $f(P,B,h)$.

Similarly, the (NC) solution can be specified in two parts, $(P^{NC}\left(h\right),h)$ where h denotes the most expensive first-stage cost observation that will be funded, and $P^{NC}\left(h\right)\in\{0,1\}^{h\times k}$ is a vector consisting of implementation probabilities $p_{i,j}^{NC}(h)$ which specifies the probability of implementing a project with Stage 2 cost m_{j} given a Stage 1 cost c_{i}. The (NC) framework assumes that the principal cannot commit to a second-stage implementation rule and therefore acts sequentially rational, which implies that she will choose the second-stage implementation independent of the first-stage cost; i.e., $p_{i,j}^{NC}\left(h\right)=p_{i',j}^{NC}\left(h\right)$ for $i,i'\in\{1,\ldots,n\}$. However, it may be the case that $p_{i,j}^{NC}(h)\ne p_{i,j}^{NC}(v)$ when $h\ne v$. Lemma 1 however confirms that $p_{i,j}^{NC}\left(h\right)$ is constant in h for $h\in\{1,\ldots,k\}$; that is, the optimal implementation of any project with continuation cost m_{j} in the (NC) problem is independent of both the first-stage cost and the total number of funded first-stage projects. Further, by sequential rationality, the (NC) solution will implement all projects with a continuation cost $m_{j}\le R$ with probability 1, and will reject all costlier projects. As was the case in the First-Best setting, we can define a single probability of implementation in the (NC) model, $p_{NC}\in\left[0,1\right]$, where $p_{NC}=\sum_{j=1}^{k}\rho_{j}p_{i,j}^{NC}\in\left[0,1\right]$ for arbitrary i. The total expected profit in the (NC) model given a threshold Stage 1 cost, c_{h}, is therefore given by $a(p_{NC},h)=\frac{h}{n}\left(E[\gamma(p_{NC})]-c_{h}\right)$.

In order to establish the result, we will show that the marginal profit of funding a manager with cost c_{h+1} in our setting with commitment is weakly greater than the marginal profit obtained in the (NC) setting. That is, we will show that:

$$f\left(P^*\left(h+1\right),B^*\left(h+1\right),h+1\right)-f\left(P^*\left(h\right),B^*\left(h\right),h\right)\geq a(p_{NC},h+1)-a(p_{NC},h).$$

To do so, we begin with the solution $\left(P^*\left(h\right),B^*\left(h\right)\right)$, and use it to generate a feasible $\left(\dot{P}\left(h+1\right),\dot{B}\left(h+1\right)\right)$ contract to the $\max_{P(h+1),B(h+1)}f\left(P\left(h+1\right),B\left(h+1\right),h+1\right)$ problem, such that $f\left(\dot{P}\left(h+1\right),\dot{B}\left(h+1\right),h+1\right)-f\left(P^*\left(h\right),B^*\left(h\right),h\right)\geq a\left(p_{NC},h+1\right)-a\left(p_{NC},h\right)$. The approach is sufficient, as $f(P^*\left(h+1\right),B^*\left(h+1\right),h+1)\geq f(\dot{P}\left(h+1\right),\dot{B}\left(h+1\right),h+1)$ by definition.

Begin by noting that $a(p_{NC},h+1)-a(p_{NC},h)=\dfrac{E[\gamma(p_{NC})-c_{h+1}-h\delta]}{n}$, as all managers with cost $c_i<c_{h+1}$ now earn an additional rent of δ dollars. In what follows, $P^*\equiv P^*(h)$, $B^*\equiv B^*(h)$ and let $\left\{\dot{P}\left(h+1\right),\dot{B}\left(h+1\right)\right\}$ be the contract generated in the following algorithm.

Algorithm

The general technique is to find a feasible contract $\left\{P,B\right\}\in[0,1]^{h+1}\times[0,\infty)^{h+1}$, such that the principal's marginal revenue obtained via $\left\{P,B\right\}$ over that obtained under $\left\{P^*,B^*\right\}\in[0,1]^h\times[0,\infty)^h$ matches the incremental expected revenue obtained in the optimal (NC) setting when the Stage 1 cost threshold is increased by one. We then show that the incremental cost in the (NC) setting of going from h to $h+1$ funded Stage 1 research projects is greater than that incurred in our setting with commitment, hence the optimal Stage 1 threshold in our setting is weakly greater than in the (NC) setting. In the (NC) setting, the marginal probability of project implementation from going from h to $h+1$ is p_{NC}, so we will generate a new contract in our setting whereby the total project implementation probability is raised by p_{NC}.

In our setting, the additional manager with cost c_{h+1} cannot have a project implemented with probability p_{NC} unless $p_{NC}\leq p_h^*$, for otherwise, the monotonicity required to satisfy the incentive compatibility constraint is violated. Thus, the general approach is to set the probability of project implementation for a manager reporting c_{h+1}, to p_h^*, and then raise the probability of implementation for a manager with lower cost from p_j^* to $\min\{p_{j-1}^*,p_{NC}\}$. This process continues until the total marginal probability of project implementation with the algorithm generated solution is equal to p_{NC}. For example, if $p_{NC}=0.7$, and $P^*=\{.9,.8,.5,.4\}$, then the algorithm will generate $P=\{.9,.8,.7,.5,.4\}$, thus the additional probability of implementation attained via P, is given by $(.7-.5)+(.5-.4)+.4=.7$, which is equal to p_{NC}. In the event that this process is insufficient; e.g. if $p_{NC}=.7$, and $P^*=\{.3,.2\}$, then the algorithm will proceed as before, but then set $p_1 = p_{NC}$; i.e., in this setting, it would generate $P = \{.7, .3, .2\}$ such that again, the incremental probability of implementation is given by $(.7 - .3) + (.3 - .2) + .2 = .7$. Because all Stage 1 cost observations are equally likely, spreading the incremental probability of implementation across multiple Stage 1 cost observations does not cause the expected incremental revenue to differ from the case where a single cost observation has a project implemented with probability p_{NC}.

By changing the implementation probabilities, the algorithm must also take into account any change in rents. If a manager experiences an increased probability of implementation, then his budget is adjusted downwards to assure that he earns the same rents as he did prior to the augmentation. If a manager with higher cost has an increased probability of implementation (as is always the case when p_{h+1} goes from 0 to $p_h > 0$), then all managers with lower costs must receive larger budgets, which in turn causes them to earn greater rents. The algorithm chooses a new budget vector such that all upward IC constraints continue to bind with one exception: If the algorithm raises the probability of implementation, p_1 to p_{NC}, then we do not reduce the rents paid to the manager reporting a cost of c_1, as we already attain the necessary (NC) marginal cost threshold. The algorithm contains three different steps and stops at either Step 1, 2 or 3. Following the algorithm, we explore the case in which it stops at each step separately, and respectively.

Step 1: Set $P_{(z)} = \left\{ p_1^*, p_2^*, ..., p_{h-1}^*, p_h^*, \min\left\{ p_{NC}, p_h^* \right\} \right\}$,

$$B_{(z)} = \left(b_1^* + \delta \frac{\min\left\{ p_{NC}, p_h^* \right\}}{p_1^*}, b_2^* + \delta \frac{\min\left\{ p_{NC}, p_h^* \right\}}{p_2^*}, ..., b_h^* + \delta \frac{\min\left\{ p_{NC}, p_h^* \right\}}{p_h^*}, c_{h+1} \right)$$ and $z = 0$.

If $p_h^* \ge p_{NC}$, then stop. If $p_h^* < p_{NC}$, and $p_1^* = p_2^* = \ldots = p_h^*$, then proceed to Step 3. Finally, if $p_h^* < p_{NC}$, and $p_1^* > p_h^*$, then let $s = s_{(z)}$, where $s_{(z)}$ denotes the largest index, i, such that $p_i^* > p_h^*$ and proceed to Step 2.

Step 2: Let $P_{(z+1)} = \left\{ p_{(z)_1}, p_{(z)_2}, \ldots, p_{(z)_s}, \min\{p_{(z)_s}, p_{NC}\}, p_{(z)_{s+2}}, \ldots, p_{(z)_{h+1}} \right\}$ and $\varepsilon_{s+1} = \min\{p_{(z)_s}, p_{NC}\} - p_{(z)_{s+1}}$; i.e., $P_{(z+1)} = P_{(z)} + e_{s+1}\varepsilon_{s+1}$ where e_{s+1} is the $s+1^{\text{th}}$ row of the $(h+1)\times(h+1)$ identity matrix. Define: $b_{(z+1)i} = b_{(z)i} + \alpha_{(z)i}$, where:

$$\alpha_{(z)_i} = \begin{cases} 0 & i > s+1 \\ -(b_{(z)_{s+1}} - c_{s+1}) \dfrac{\varepsilon_{s+1}}{p_{(z)_{s+1}} + \varepsilon_{s+1}} & i = s+1 \\ \delta \dfrac{\varepsilon_{s+1}}{p_{(z)_i}} & i < s+1 \end{cases}$$

If $p_{(z+1)_{s+1}} = p_{NC}$, then stop. If $p_{(z+1)s+1} < p_{NC}$ and $p_{(z+1)s+1} < p_1^*$, then let s_{z+1} define the largest index i such that $p_i^* > p_{(z+1)s+1}$, set $z = z+1$, $s = s_{(z+1)}$ and reiterate Step 2. If $p_{(z+1)_{s+1}} = p_1^* < p_{NC}$, then set $z = z+1$ and proceed to Step 3.

Step 3: Set $P_{(z+1)} = \left\{ p_{AE}, p_{{}_{(z)}2}, \dots, p_{(z)_{h+1}} \right\}$, $b_{(z+1)} = b_{(z)}$, and stop.

Sufficiency

We first show that the contract $\left\{ P_{(z)}, B_{(z)} \right\}$ satisfies all incentive compatibility (IC) and individual rationality (IR) constraints, after which we show that this contract yields the same marginal revenue as that obtained in the (NC) framework, albeit at (weakly) lower cost. We consider the three following cases: when the algorithm stops at Step 1 ($p_h^* \ge p_{NC}$), when it stops at Step 2 ($p_1^* \ge p_{NC} > p_h^*$), and finally, when it stops at Step 3 ($p_{NC} > p_1^*$).

Case 1: If the algorithm stops at Step 1, then the proposed solution is given by $P_{(0)} = \left\{ p_1^*, \dots, p_h^*, p_{NC} \right\}$, $b_{(0)_i} = b_i^* + \delta \dfrac{p_{NC}}{p_i^*}$ for $i < h+1$, and $b_{(0)_{h+1}} = c_{h+1}$, with $p_{NC} \le p_h^*$. The manager with reported cost c_{h+1} is paid his reservation utility, and therefore his individual rationality constraint is satisfied. A manager with reported cost c_h is now paid $b_h^* + \delta \dfrac{\min\{p_{NC}, p_h^*\}}{p_h^*} = c_h + \delta \dfrac{p_{NC}}{p_h^*}$, therefore his IR constraint continues to hold and his incentive compatibility constraint is $\left(c_h + \dfrac{p_{NC}}{p_h^*}\delta - c_h \right) p_h^* \ge (c_{h+1} - c_h) p_{NC} = (c_h + \delta - c_h) p_{NC}$. But since $\left(c_h + \dfrac{p_{NC}}{p_h^*}\delta - c_h \right) p_h^* = p_{NC}\delta$, the manager with reported cost c_h has his IC constraint satisfied with equality. Recall that under $\left(P^*, B^* \right)$, a manager with reported cost c_i with $i < h$ earned a rent of $\sum_{s=i+1}^{h} \delta \dfrac{p_s^*}{p_i^*}$. In particular, $b_i^* = c_i + \sum_{s=i+1}^{h} \delta \dfrac{p_s^*}{p_i^*}$. Since $b_{(0)_i} = b_i^* + \delta \dfrac{p_{NC}}{p_i^*}$, we can express the IC constraint for a manager with reported cost, c_i with $i < h$ as any of following equivalent statements:

$$
\begin{aligned}
(b_{(0)_i}^* - c_i) p_i^* &\ge (b_{(0)_{i+1}} - c_i) p_{i+1}^* \\
(b_i^* + \frac{\delta p_{NC}}{p_i^*} - c_i) p_i^* &\ge (b_{i+1}^* + \delta \frac{p_{NC}}{p_{i+1}^*} - c_i) p_{i+1}^* \\
(b_i^* - c_i) p_i^* + \delta p_{NC} &\ge (b_{i+1}^* - c_i) p_{i+1}^* + \delta p_{NC} \\
(b_i^* - c_i) p_i^* &\ge (b_{i+1}^* - c_i) p_{i+1}^*.
\end{aligned}
$$

The last inequality above is valid, as it held for the original solution pair, $\left\{P^*, B^*\right\}$. The expected incremental cost to the principal of $\left\{P_{(0)}, B_{(0)}\right\}$ versus $\left\{P^*, B^*\right\}$ is: $\frac{1}{n}\left(c_{h+1} + \delta \sum_{i=1}^{h} \frac{p_{NC}}{p_i^*}\right)$, yet because $p_{NC} \le p_h^* \le p_i^*$, the incremental expected cost is bounded above by $\frac{c_{h+1} + h\delta}{n}$ which is the expected incremental cost in the (NC) framework. The expected revenue obtained under $\left\{P_{(0)}, B_{(0)}\right\}$ net of that obtained with $\left\{P^*, B^*\right\}$ is given by $\frac{1}{n}E[\gamma(p_{NC})]$, which is the same as that found in the (NC) setting. Thus, the expected profit with $\left\{P_{(0)}, B_{(0)}\right\}$ is weakly greater than that under the (NC) scheme, and strictly greater as long as $p_1^* > p_h^*$, as was to be shown.

Case 2: Suppose that the algorithm stops at Step 2. Before proceeding with the general case, we first verify that all constraints are satisfied and the incremental probability of implementation from the (NC) setting is reached after a single iteration of Step 2; i.e., when the algorithm has stopped with $z = 0$ (though the algorithm has generated a candidate solution, $\left\{P_{(1)}, B_{(1)}\right\}$). To do so, we examine three possible alternatives. First, (a), we examine the alternative in which a manager has observed a cost c_i where $i > s_{(0)} + 1$. Next, (b), we consider the situation where the manager observes a cost of $c_i = c_{s_{(0)}+1}$, and finally, (c), that the manager observes a cost observation c_i, where $i < s_{(0)} + 1$.

(a) As $p_{(1)_i} = p_{(0)_i}$ and $b_{(1)_i} = b_{(0)_i}$ for $i > s_{(0)} + 1$, all constraints are unchanged and remain satisfied.

(b) The expected utility to a manager truthfully reporting cost $c_{s_{(0)}+1}$ is now given by:

$$
\begin{aligned}
&\left(b_{(1)_{s_{(0)}+1}} - c_{s_{(0)}+1}\right) p_{(1)_{s_{(0)}+1}} \\
&\quad = \left(b_{(0)_{s_{(0)}+1}} + \alpha_{(0)_{s_{(0)}+1}} - c_{s_{(0)}+1}\right)\left(p_{s_{(0)}+1} + \varepsilon_{s_{(0)}+1}\right) \\
&\quad = \left(b_{(0)_{s_{(0)}+1}} - c_{s_{(0)}+1} + (b_{(0)_{s_{(0)}+1}} - c_{s_{(0)}+1}) \frac{-\varepsilon_{s_{(0)}+1}}{p_{(0)_{s_{(0)}+1}} + \varepsilon_{s_{(0)}+1}}\right)\left(p_{s_{(0)}+1} + \varepsilon_{s_{(0)}+1}\right) \\
&\quad = \left((b_{(0)_{s_{(0)}+1}} - c_{s_{(0)}+1}) \frac{p_{(0)_{s_{(0)}+1}} + \varepsilon_{s_{(0)}+1} - \varepsilon_{s_{(0)}+1}}{p_{(0)_{s_{(0)}+1}} + \varepsilon_{s_{(0)}+1}}\right)\left(p_{s_{(0)}+1} + \varepsilon_{s_{(0)}+1}\right) \\
&\quad = \left(b_{(0)_{s_{(0)}+1}} - c_{s_{(0)}+1}\right) p_{(0)_{s_{(0)}+1}} \frac{p_{s_{(0)}+1} + \varepsilon_{s_{(0)}+1}}{p_{(0)_{s_{(0)}+1}} + \varepsilon_{s_{(0)}+1}} \\
&\quad = (b_{(0)s_{(0)}+1} - c_{s_{(0)}+1}) p_{(0)s_{(0)}+1}.
\end{aligned}
$$

Thus, a manager with reported cost $c_{s_{(0)}+1}$ and contract $\left\{P_{(1)}, B_{(1)}\right\}$ earns exactly the same rent as under $\left\{P_{(0)}, B_{(0)}\right\}$, and thus, his IR constraint and IC constraint are satisfied since they were shown to be satisfied under $\left\{P_{(0)}, B_{(0)}\right\}$, and all managers with greater cost have the exact same contract under $\left\{P_{(1)}, B_{(1)}\right\}$ as they did under $\left\{P_{(0)}, B_{(0)}\right\}$.

(c) Finally, consider a manager whose cost is c_i where $i < s_0 + 1$. His expected utility for truthfully reporting his cost is now given by:

$$\begin{aligned} \left(b_{(1)_i} - c_i\right) p_{(1)_i} &= \left(b_{(0)_i} + \alpha_{(0)_i} - c_i\right) p_{(0)_i} \\ &= \left(b_{(0)_i} + \delta \frac{\varepsilon_{s_{(0)}+1}}{p_{(0)_i}} - c_i\right) p_{(0)_i} \\ &= \left(b_{(0)_i} - c_i\right) p_{(0)_i} + \delta\varepsilon_{s_{(0)}+1}. \end{aligned}$$

If manager c_i reports c_{i+1} and $i+1 = s_{(0)} + 1$ then he expects to earn

$$\begin{aligned} (b_{(1)_{i+1}} - c_i) p_{(1)_{i+1}} &= (b_{(1)_{i+1}} - c_{i+1} + \delta) p_{(1)_{i+1}} \\ &= \left(b_{(0)_{i+1}} + \alpha_{(0)_{i+1}} - c_{i+1}\right) p_{(1)_{i+1}} + \delta p_{(1)_{i+1}} \\ &= \left(b_{(0)_{i+1}} - c_{i+1} + (b_{(0)_{i+1}} - c_{i+1}) \frac{-\varepsilon_{i+1}}{p_{(0)_{i+1}} + \varepsilon_{i+1}}\right)\left(p_{(0)_{i+1}} + \varepsilon_{i+1}\right) + \delta p_{(1)_{i+1}} \\ &= (b_{(0)_{i+1}} - c_{i+1}) p_{(0)_{i+1}} + \delta p_{(1)_{i+1}}. \end{aligned}$$

On the other hand, if he reports c_{i+1} where $i+1 < s_{(0)} + 1$, he expects to earn:

$$\begin{aligned} (b_{(1)_{i+1}} - c_i) p_{(1)_{i+1}} &= (b_{(0)_{i+1}} + \delta \frac{\varepsilon_{s_{(0)}+1}}{p_{(0)_{i+1}}} - c_i) p_{(0)_{i+1}} \\ &= (b_{(0)_{i+1}} - c_i) p_{(0)_{i+1}} + \delta\varepsilon_{s(0)+1}. \end{aligned}$$

Now, if $i+1 = s_{(0)} + 1$, then manager i's IC constraint is satisfied if any on the following equivalent statements hold:

$$\begin{aligned} \left(b_{(0)_i} - c_i\right) p_{(0)_i} + \delta\varepsilon_{s_{(0)}+1} &\geq \left(b_{(0)_{i+1}} - c_{i+1}\right) p_{(0)_{i+1}} + \delta p_{(1)_{i+1}} \\ \left(b_{(0)_i} - c_i\right) p_{(0)_i} + \delta\varepsilon_{s_{(0)}+1} &\geq \left(b_{(0)_{i+1}} - c_{i+1} + \delta\right) p_{(0)_{i+1}} + \delta\left(p_{(1)_{i+1}} - p_{(0)_{i+1}}\right) \\ \left(b_{(0)_i} - c_i\right) p_{(0)_i} + \delta\varepsilon_{s_{(0)}+1} &\geq \left(b_{(0)_{i+1}} - c_i\right) p_{(0)_{i+1}} + \delta\left(\varepsilon_{i+1}\right) \\ \left(b_{(0)_i} - c_i\right) p_{(0)_i} &\geq \left(b_{(0)_{i+1}} - c_i\right) p_{(0)_{i+1}}. \end{aligned}$$

Whereas if: $i+1<s_{(0)}+1$, then manager i's IC constraint is satisfied if any on the following equivalent statements hold:

$$\left(b_{(0)_i}-c_i\right)p_{(0)_i}+\delta\varepsilon_{s_{(0)}+1}\geq(b_{(0)_{i+1}}-c_i)p_{(0)_{i+1}}+\delta\varepsilon_{s_{(0)}+1}$$

$$\left(b_{(0)_i}-c_i\right)p_{(0)_i}\geq(b_{(0)_{i+1}}-c_i)p_{(0)_{i+1}}.$$

In both cases, the final statement holds, because the IC constraint was satisfied under $\left\{P_{(0)},B_{(0)}\right\}$. Since the algorithm did not proceed to Step 3, we know that $p_{NC}=p_{(1)_{s_{(0)}+1}}$.

Given that the (IC) and (IR) constraints in our solution are satisfied we next compare the incremental expected revenues and costs of increasing the Stage 1 threshold by one versus doing so in the NC setting. In our proposed solution, a manager reporting cost c_{h+1} generates an expected revenue of $\frac{1}{n}E[\gamma(p_h^*)]$, and a manager reporting a cost of $c_{s_{(0)}+1}$ generates an incremental expected revenue of $\frac{1}{n}\left(E[\gamma(p_h^*+\varepsilon_{s_{(0)}+1})]-E[\gamma(p_h^*)]\right)$. Since the probability of a c_{h+1} cost manager is identical to that of a $c_{s_{(0)}+1}$ manager, the incremental expected revenue is given by $\frac{1}{n}E[\gamma(p_h^*+\varepsilon_{s_{(0)}+1})]=\frac{1}{n}E[\gamma(p_{NC})]$, the same level as in the NC problem. Note that the principal's incremental expected cost after Step 1 was $\frac{1}{n}\left(c_{h+1}+\delta\sum_{i=1}^{h}\frac{p_h^*}{p_i^*}\right)$. In this single iteration of Step 2: we have raised the payments to managers reporting a cost c_i with $i<s_{(0)}+1$ by $\alpha_i=\delta\frac{\varepsilon_{s_{(0)}+1}}{p_i^*}$, hence the total incremental expected costs after Step 1 and Step 2 are given by (note we have omitted the $1/n$ pre-multiplier below):

$$\left(\begin{array}{c} c_{h+1}+\delta\sum\limits_{i=1}^{h}\frac{p_h^*}{p_i^*}+\delta\sum\limits_{i=1}^{s_{(0)}}\frac{\varepsilon_{s_{(0)}+1}}{p_i^*} \\ \underbrace{-(b_{(0)s_{(0)}+1}-c_{s_{(0)}+1})\frac{\varepsilon_{s_{(0)}+1}}{p_{(0)s_{(0)}+1}+\varepsilon_{s_{(0)}+1}}}_{>0}\end{array}\right)<c_{h+1}+\delta\sum_{i=1}^{h}\frac{p_h^*}{p_i^*}+\delta\sum_{i=1}^{s_{(0)}}\frac{\varepsilon_{s_{(0)}+1}}{p_i^*}$$

$$=c_{h+1}+\delta\sum_{i=s_{(0)}+1}^{h}\frac{p_h^*}{p_i^*}+\delta\sum_{i=1}^{s_{(0)}}\left(\frac{p_h^*}{p_i^*}+\frac{\varepsilon_{s_{(0)}+1}}{p_i^*}\right)$$

$$=c_{h+1}+\delta\sum_{i=s_{(0)}+1}^{h}\frac{p_h^*}{p_i^*}+\delta\sum_{i=1}^{s_{(0)}}\left(\frac{p_h^*}{p_i^*}+\frac{p_{NC}-p_h^*}{p_i^*}\right)$$

$$=c_{h+1}+\delta\sum_{i=s_{(0)}+1}^{h}\frac{p_h^*}{p_i^*}+\delta\sum_{i=1}^{s_{(0)}}\left(\frac{p_{NC}}{p_i^*}\right).$$

Since $p_i^* = p_h^*$ for all $i = s_{(0)} + 1, \ldots, h$, the first sum is equal to $h - s_{(0)}$. Because Step 2 ended with $z = 1$, we know that $p_{NC} \le p_{s_{(0)}}^* \le p_i^*$ for $i < s_{(0)}$, therefore the second sum is bound by $s_{(0)}$, implying that the incremental expected cost is no larger than the NC incremental expected cost of $1\!\!/_{n}\left(c_{h+1} + h\delta\right)$.

Now, for the general case of the algorithm stopping at Step 2 after $z+1$ iterations; i.e., when $z \ge 1$. Since $p_{(z+1)_j} = p_{(z)_j}$ and $b_{(z+1)_j} = b_{(z)_j}$ for $j > s_{(z)} + 1$, all (IC) constraints are inductively satisfied for managers observing costs greater than $c_{s_{(z)}+1}$. The expected utility to a manager truthfully reporting cost $c_{s_{(z)}+1}$ is now given by:

$$
\begin{aligned}
&\left(b_{(z+1)_{s(z)+1}} - c_{s_{(z)}+1}\right) p_{(z+1)_{s(z)+1}} \\
&\quad = \left(b_{(z)_{s(z)+1}} + \alpha_{(z)_{s(z)+1}} - c_{s_{(z)}+1}\right) p_{(z+1)_{s(z)+1}} \\
&\quad = \left(b_{(z)_{s(z)+1}} - c_{s_{(z)}+1} + (b_{(z)_{s(z)+1}} - c_{s_{(z)}+1}) \frac{-\varepsilon_{s_{(z)}+1}}{p_{(z)_{s(z)+1}} + \varepsilon_{s_{(z)}+1}} \right) \left(p_{(z)_{s_{(z)}+1}} + \varepsilon_{s_{(z)}+1} \right) \qquad (2) \\
&\quad = \left((b_{(z)_{s(z)+1}} - c_{s_{(z)}+1}) \frac{p_{(z)s_{(z)}+1}}{p_{(z)_{s(z)+1}} + \varepsilon_{s_{(z)}+1}} \right) \left(p_{(z)_{s_{(z)}+1}} + \varepsilon_{s_{(z)}+1} \right) \\
&\quad = \left(b_{(z)_{s(z)+1}} - c_{s_{(z)}+1}\right) p_{(z)_{s(z)+1}}.
\end{aligned}
$$

The expected utility to a manager truthfully reporting $c_{s_{(z)}+1}$ is thus the same as he earned under $\left\{P_{(z)}, B_{(z)}\right\}$, which is strictly greater than that earned under $\left\{P^*, B^*\right\}$ because in previous iterations, his rent was raised to accommodate the increased probability of implementation for costlier types. As such, his IR constraint is satisfied. Because the $z+1^{\text{th}}$ iteration of Step 2 does not affect the contracts facing managers with greater cost than $c_{s_{(z)}+1}$, manager $s_{(z)}+1$'s (IC) constraint is satisfied as well.

Next consider the (IC) constraint for a manager with cost observation c_i where $i < s_{(z)} + 1$, we first consider his payoff to reporting $i+1$ if $i = s_{(z)}$ in which case he expects to earn:

$$
\begin{aligned}
(b_{(z+1)_{s(z)+1}} - c_{s_{(z)}}) p_{(z+1)_{s(z)+1}} &= (b_{(z+1)_{s(z)+1}} - c_{s_{(z)}+1} + \delta) p_{(z+1)_{s(z)+1}} \\
&= (b_{(z+1)_{s(z)+1}} - c_{s_{(z)}+1}) p_{(z+1)_{s(z)+1}} + \delta p_{(z+1)_{s(z)+1}} \\
&= (b_{(z)_{s(z)+1}} - c_{s_{(z)}+1}) p_{(z)_{s(z)+1}} + \delta p_{(z+1)_{s(z)+1}},
\end{aligned}
$$

where the last equality follows from (2). In this case the manager's IC constraint is satisfied if any on the following equivalent statements hold:

$$\left(b_{(z)_i}-c_i\right)p_{(z)_i}+\delta\varepsilon_{s_{(z)}+1}\geq(b_{(z)_{i+1}}-c_{i+1})p_{(z)_{i+1}}+\delta p_{(z+1)_{i+1}}$$
$$\left(b_{(z)_i}-c_i\right)p_{(z)_i}+\delta\varepsilon_{s_{(z)}+1}\geq(b_{(z)_{i+1}}-c_{i+1}+\delta)p_{(z)_{i+1}}+\delta(p_{(z+1)_{i+1}}-p_{(z)_{i+1}})$$
$$\left(b_{(z)_i}-c_i\right)p_{(z)_i}+\delta\varepsilon_{s_{(z)}+1}\geq(b_{(z)_{i+1}}-c_{i+1}+\delta)p_{(z)_{i+1}}+\delta(\varepsilon_{i+1})$$
$$\left(b_{(z)_i}-c_i\right)p_{(z)_i}\geq(b_{(z)_{i+1}}-c_{i+1}+\delta)p_{(z)_{i+1}}$$
$$\left(b_{(z)_i}-c_i\right)p_{(z)_i}\geq(b_{(z)_{i+1}}-c_i)p_{(z)_{i+1}}.$$

The final statement above in terms of (z) holds, since we arrived at (z) iteratively from $z=0$, which was proved earlier.

On the other hand, if manager c_i reports c_{i+1} where $i<s_{(z)}$, then he expects to earn:

$$(b_{(z+1)_{i+1}}-c_i)p_{(z+1)_{i+1}}=(b_{(z)_{i+1}}+\delta\frac{\varepsilon_{s_{(z)}+1}}{p_{(z)_{i+1}}}-c_i)p_{(z+1)_{i+1}}=(b_{(z)_{i+1}}+\delta\frac{\varepsilon_{s_{(z)}+1}}{p_{(z)_{i+1}}}-c_i)p_{(z)_{i+1}}=(b_{(z)_{i+1}}-c_i)p_{(z)_{i+1}}+\delta\varepsilon_{s_{(z)}+1},$$

and manager i's (IC) constraint is satisfied if any on the following equivalent statements hold:

$$\left(b_{(z)_i}-c_i\right)p_{(z)_i}+\delta\varepsilon_{s_{(z)}+1}\geq\left(b_{(z)_{i+1}}-c_i\right)p_{(z)_{i+1}}+\delta\varepsilon_{s_{(z)}+1}$$
$$\left(b_{(z)_i}-c_i\right)p_{(z)_i}\geq\left(b_{(z)_{i+1}}-c_i\right)p_{(z)_{i+1}}.$$

Again, we find that the last inequality above holds inductively for (z), thus we have shown that all (IC) constraints are satisfied at the $(z+1)^{\text{th}}$ iteration of Step 2.

Next consider the incremental expected costs and revenues. Now, recall that the incremental expected cost at Step 1 was given by $\frac{1}{n}c_{h+1}+\delta\sum_{i=1}^{h}\frac{p_h^*}{p_i^*}$. At the first iteration of Step 2, we added an additional cost of $\frac{1}{n}\sum_{s=1}^{s(0)}\frac{\varepsilon_{s(0)+1}}{p_s^*}$, at the second iteration, we added $\frac{1}{n}\sum_{s=1}^{s(1)}\frac{\varepsilon_{s(1)+1}}{p_s^*}$ and so on. This provides us with the following expression for the incremental expected cost of funding cost observation c_{h+1} in our setting ($1/n$ pre-multiplier omitted):

$$\left(\begin{array}{l}\delta\left(\sum_{s=1}^{s_{(z)}}\frac{\varepsilon_{s_{(0)}+1}+\ldots+\varepsilon_{s_{(z)}+1}+p_h^*}{p_s^*}+\sum_{s=s_{(z)}+1}^{s_{(z-1)}}\frac{\varepsilon_{s_{(0)}+1}+\ldots+\varepsilon_{s_{(z-1)}+1}+p_h^*}{p_{(z-1)_s}}+\ldots+\sum_{s=s_{(0)}+1}^{h}\frac{p_h^*}{p_{(0)_s}}\right)\\ +c_{h+1}-\underbrace{\sum_{s=s_{(0)}}^{s_{(z)}}(b_{(z)_{s+1}}-c_{s+1})\frac{\varepsilon_{s+1}}{p_{(z)_{s+1}}+\varepsilon_{s+1}}}_{>0}\end{array}\right)$$
$$<c_{h+1}+\delta\left(\sum_{s=1}^{s_{(z)}}\frac{\varepsilon_{s_{(0)}+1}+\ldots+\varepsilon_{s_{(z)}+1}+p_h^*}{p_s^*}+\sum_{s=s_{(z)}+1}^{s_{(z-1)}}\frac{\varepsilon_{s_{(0)}+1}+\ldots+\varepsilon_{s_{(z-1)}+1}+p_h^*}{p_{(z-1)_s}}+\ldots+\sum_{s=s_{(0)}+1}^{h}\frac{p_h^*}{p_{(0)_s}}\right).$$

If $p_{s_{(i)}}<p_{NC}$, then we can use $\varepsilon_{s+1}=p_s^*-p_{s+1}^*$ to re-express the numerators in the above expression in

the following way: $p_h^* + \sum_{s=s_{(0)}}^{s_{(i)}} \varepsilon_{s+1} = p_h^* + \sum_{s=s_{(0)}}^{s_{(i)}} \left(p_s^* - p_{s+1}^*\right) = p_{s_{(i)}}{}^*$, where the last equality holds because $p_{s_{(0)}+1} = p_h^*$ by construction. This simplification will facilitate the calculation of the total marginal cost below. We can now express the right hand side of the incremental cost above as:

$$c_{h+1} + \delta\left(\sum_{s=1}^{s_{(z)}} \frac{p_{NC}}{p_s^*} + \sum_{s=s_{(z)}+1}^{s_{(z-1)}} \frac{p_{s_{z-1}}^*}{p_{(z-1)_s}} + \ldots + \sum_{s=s_{(0)}+1}^{h} \frac{p_h^*}{p_s^*}\right).$$

This shows that at each iteration, all managers with cost $c_i \le c_{s_{(z)}}$ earn an additional rent of $\delta \frac{p_{s_{(z)}}}{p_i^*}$, thus after $z+1$ iterations, our total marginal cost is bounded by the sum of the new rents, plus the reimbursement of c_{h+1} for the newly funded manager. We can further simplify the bound on our incremental cost to:

$$c_{h+1} + \delta\left(\sum_{s=s_{(1)}}^{s_{(z)}} \frac{p_{NC}}{p_s} + (s_{(z-1)} - s_{(z)}) + \ldots + (h - s_{(0)})\right) = c_{h+1} + \delta\left(\sum_{s=s_{(1)}}^{s_{(z)}} \frac{p_{NC}}{p_s} + h - s_{(z-1)}\right) \le c_{h+1} + h\delta.$$

We note that the equality holds because after z iterations of Step 2, all probabilities $p_{(z-1)s} = p_s^*$ for $s \le s_{(z-1)}$, as these probabilities have not yet been subject to change. The last inequality holds, because $p_{NC} \le p_{s_{(z)}}$. Since the last expression above is equal to the incremental expected cost in the NC framework, all that remains is to verify that the (NC) incremental expected revenues equal those of the proposed solution. To do so, note that after the first iteration of Step 2; the principal earns an incremental expected revenue of $\frac{1}{n}\left(E[\gamma(p_{s_{(0)}})] - E[\gamma(p_h^*)]\right)$, the second iteration raises a marginal $\frac{1}{n}\left(E[\gamma(p_{s_{(1)}})] - E[\gamma(p_{s_{(0)}})]\right)$, and so one. Thus after $z+1$ iterations of Step 2, the principal will have raised $\frac{1}{n}\left(E[\gamma(p_{NC})] - E[\gamma(p_h^*)]\right)$ revenues, which, paired with the marginal revenue raised at Step 1, $\frac{1}{n}E[\gamma(p_h^*)]$, earns her an incremental expected revenue of $\frac{1}{n}E[\gamma(p_{NC})]$, which is the same as that obtained in the NC setting, as was to be shown.

Case 3: If the algorithm is completed at Step 3, then either Step 2 was bypassed, or we have passed through, say, z iterations of Step 2. We begin with the latter case. The final iteration of Step 2 generates a feasible pairing $\{P_{(z)}, B_{(z)}\}$, where the incremental expected cost to the principal is given by:

$$\frac{1}{n}\left(c_{h+1} + \delta\left(\sum_{s=1}^{s_{(z)}} \frac{p_1^*}{p_s^*} + h - s_{(z-1)}\right)\right).$$

Since we have moved to Step 3, $p_i^* = p_1^*$ for $i = 1, \ldots, s_{(z)}$, hence the incremental expected cost is identical to that in the NC framework. The incremental expected revenue raised by the end of Step 2 (after Step 1) is $\frac{1}{n}E[\gamma(p_1^*)]$. In Step 3, the probability of having a project implemented for a manager reporting a cost of c_1 is raised, hence his rents are raised although the principal has not changed the budget $b_{(z)_1}$, implying that the contract generated at Step 3, $\left\{P_{(z+1)}, B_{(z+1)}\right\}$, continues to satisfy all IR and IC constraints. Moreover, Step 3 generates an incremental expected revenue of $\frac{1}{n}\left(E[\gamma(p_{NC})] - E[\gamma(p_1^*)]\right)$, hence the revenue attained at the end of Step 3 matches that raised under the NC setting.

On the other hand, if Step 2 was bypassed, then $p_1^* = \ldots = p_h^*$ and Step 3 generates a contract $\left\{P_{(1)}, B_{(1)}\right\}$ with $P_{(1)} = \left\{p_{AE}, p_{(0)_2}, \ldots, p_{(0)_{h+1}}\right\}$ and $B_{(1)} = B_{(0)}$. The only difference between this proposed solution and $\left\{P_{(0)}, B_{(0)}\right\}$ which emerged from Step 1, is the increased probability of implementation facing the manager who reports c_1. In particular, all (IC) and (IR) constraints are satisfied for all managers with a reported cost in excess of c_1 and the (IC) constraint facing manager c_1 is now strengthened; as he now earns greater expected rents via truthful disclosure than he did previously, in spite of the budget $b_{(0)_1}$ being unchanged. Because the principal's outlays are unchanged, there are no additional rents or costs paid, hence, the incremental expected revenue and cost associated in this sub-case of Case 3 is identical to that discussed above, completing the proof.

Lemma 1: *We claim that* $p_{NC}(h) = \arg\max_p h\mathrm{E}\left[\gamma(p)\right] - hc_h = p_{NC}(j)$ *for* $j \in \{1, \ldots, k\}$*. That is, the (NC) optimal implementation probability is independent of the number of projects funded in the first stage.* Proof is trivial and therefore omitted.

Proposition 5:

Because we have simplified the principal’s problem to solving for the vector $P^* = \left\{p_1^*, p_2^*, \ldots, p_h^*\right\}$, we first establish that the principal's objective function, f, satisfies super-modularity and increasing-differences in all choice variables, p_j. A sufficient condition is for all cross-partials, $\frac{\partial^2 f(P)}{\partial p_i \partial p_j}$ and $\frac{\partial^2 f}{\partial p_i \partial G_j}$ with $i \neq j$ to carry the same sign (Sundaram [2009]). Because:

$$\frac{\partial^2 f}{\partial p_i \partial p_j} = \begin{cases} \dfrac{\delta}{p_j^2} & j < i \\ \dfrac{\delta}{p_i^2} & j > i \end{cases}$$

we conclude that f is super-modular over $X^h \subseteq \mathbb{R}^h$. Further, because $\frac{\partial^2 f}{\partial p_i \partial G_j} = 1 > 0$, the function f is super-modular over $P^* \times \{G_1, G_2, \ldots, G_k : G_i < G_{i+1}\}$, as was to be shown.

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